CORRECTION SÉRIE LIMITE D'UNE FONCTION NUMÉRIQUE

EXERCICE 1 .

Calculons les limites suivantes :

- $\lim\limits_{x\longrightarrow 1}\frac{3x^2-2x-1}{x-1}=\text{"}\frac{0}{0}\text{"}\quad (F.I)$

On a : $3x^2-2x-1=(3x+1)(x-1)$ *donc*

$$\begin{aligned}\lim_{x\longrightarrow 1}\frac{3x^2-2x-1}{x-1} &= \lim_{x\longrightarrow 1}\frac{3x+1\,(x-1)}{x-1}\\ &= \lim_{x\longrightarrow 1}3x+1\\ &= 3\times 1+1\\ &= 4\end{aligned}$$

- $\lim\limits_{x\longrightarrow 1}\frac{x^4-2x^3+x^2+x-1}{x^2+x-2}=\text{"}\frac{0}{0}\text{"}\quad (F.I)$

On a : $x^4-2x^3+x^2+x-1=(x-1)(x^3-x^2+1)$ *et* $x^2+x-2=(x+2)(x-1)$ *donc*

$$\begin{aligned}\lim_{x\longrightarrow 1}\frac{x^4-2x^3+x^2+x-1}{x^2+x-2} &= \lim_{x\longrightarrow 1}\frac{(x-1)(x^3-x^2+1)}{(x+2)(x-1)}\\ &= \lim_{x\longrightarrow 1}\frac{x^3-x^2+1}{x+2}\\ &= \frac{-1+1+1}{1+2}=\frac{1}{3}\end{aligned}$$

- $\lim\limits_{x\longrightarrow 0}\frac{\sqrt{4-x}-\sqrt{4+x}}{x}=\text{"}\frac{0}{0}\text{"}\quad (F.I)$

On a

$$\begin{aligned}\lim_{x\longrightarrow 0}\frac{\sqrt{4-x}-\sqrt{4+x}}{x} &= \lim_{x\longrightarrow 0}\frac{\left(\sqrt{4-x}-\sqrt{4+x}\right)\left(\sqrt{4-x}+\sqrt{4+x}\right)}{x\left(\sqrt{4-x}+\sqrt{4+x}\right)}\\ &= \lim_{x\longrightarrow 0}\frac{(4-x)-(4+x)}{x\left(\sqrt{4-x}+\sqrt{4+x}\right)}\\ &= \lim_{x\longrightarrow 0}\frac{-2x}{x\left(\sqrt{4-x}+\sqrt{4+x}\right)}\\ &= \lim_{x\longrightarrow 0}\frac{-2}{\sqrt{4-x}+\sqrt{4+x}}=\frac{-2}{\sqrt{4-0}+\sqrt{4+0}}=\frac{-1}{2}\end{aligned}$$

donc $\lim\limits_{x\longrightarrow 0}\frac{\sqrt{4-x}-\sqrt{4+x}}{x}=\frac{-1}{2}.$

- $\lim\limits_{x\longrightarrow 0}\dfrac{\sqrt{x+1}-1}{x^2-x}=\text{"}\dfrac{0}{0}\text{"}\quad (F.I)$

  On a

$$\begin{aligned}
\lim_{x\longrightarrow 0}\frac{\sqrt{x+1}-1}{x^2-x} &= \lim_{x\longrightarrow 0}\frac{\left(\sqrt{x+1}-1\right)\left(\sqrt{x+1}+1\right)}{x\,(x-1)\left(\sqrt{x+1}+1\right)}\\
&= \lim_{x\longrightarrow 0}\frac{(x+1)-1}{x\,(x-1)\left(\sqrt{x+1}+1\right)}\\
&= \lim_{x\longrightarrow 0}\frac{x}{x\,(x-1)\left(\sqrt{x+1}+1\right)}\\
&= \lim_{x\longrightarrow 0}\frac{1}{(x-1)\left(\sqrt{x+1}+1\right)}=\frac{1}{(0-1)\left(\sqrt{0+1}+1\right)}=-\frac{1}{2}
\end{aligned}$$

  donc $\lim\limits_{x\longrightarrow 0}\dfrac{\sqrt{x+1}-1}{x^2-x}=\dfrac{-1}{2}.$

EXERCICE 2 *;*

Calculons les limites suivantes :

- $\lim\limits_{x\longrightarrow 0}\dfrac{\sqrt{x+1}+\sqrt{x+4}-3}{x}=\text{"}\dfrac{0}{0}\text{"}\quad (F.I)$

  On a

$$\begin{aligned}
\lim_{x\longrightarrow 0}\frac{\sqrt{x+1}+\sqrt{x+4}-3}{x} &= \lim_{x\longrightarrow 0}\frac{\sqrt{x+1}-1+\sqrt{x+4}-2}{x}\\
&= \lim_{x\longrightarrow 0}\frac{\sqrt{x+1}-1}{x}+\frac{\sqrt{x+4}-2}{x}\\
&= \lim_{x\longrightarrow 0}\frac{\left(\sqrt{x+1}-1\right)\left(\sqrt{x+1}+1\right)}{x\left(\sqrt{x+1}+1\right)}+\frac{\left(\sqrt{x+4}-2\right)\left(\sqrt{x+4}+2\right)}{x\left(\sqrt{x+4}+2\right)}\\
&= \lim_{x\longrightarrow 0}\frac{(x+1)-1}{x\left(\sqrt{x+1}+1\right)}+\frac{x+4-4}{x\sqrt{x+4}+2}\\
&= \lim_{x\longrightarrow 0}\frac{x}{x\left(\sqrt{x+1}+1\right)}+\frac{x}{x\sqrt{x+4}+2}\\
&= \lim_{x\longrightarrow 0}\frac{1}{\sqrt{x+1}+1}+\frac{1}{\sqrt{x+4}+2}\\
&= \frac{1}{\sqrt{0+1}+1}+\frac{1}{\sqrt{0+4}+2}=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}
\end{aligned}$$

  donc $\lim\limits_{x\longrightarrow 0}\dfrac{\sqrt{x+1}+\sqrt{x+4}-3}{x}=\dfrac{3}{4}.$

- $\lim\limits_{x\longrightarrow 3}\dfrac{\sqrt{6+x}-3}{x^2-x-6}=\text{"}\dfrac{0}{0}\text{"}\quad (F.I)$

On a : $\lim\limits_{x\longrightarrow 3}\dfrac{\sqrt{6+x}-3}{x^2-x-6}=\lim\limits_{x\longrightarrow 3}\dfrac{\left(\sqrt{6+x}-3\right)\left(\sqrt{6+x}+3\right)}{\left(x^2-x-6\right)\left(\sqrt{6+x}+3\right)}$ *et comme* $x^2-x-6=(x+2)(x-3)$ *donc*

$$\begin{aligned}\lim_{x\longrightarrow 3}\frac{\left(\sqrt{6+x}-3\right)\left(\sqrt{6+x}+3\right)}{\left(x^2-x-6\right)\left(\sqrt{6+x}+3\right)} &= \lim_{x\longrightarrow 3}\frac{\left(\sqrt{6+x}-3\right)\left(\sqrt{6+x}+3\right)}{(x+2)(x-3)\left(\sqrt{6+x}+3\right)}\\ &= \lim_{x\longrightarrow 3}\frac{(6+x)-9}{(x+2)(x-3)\left(\sqrt{6+x}+3\right)}\\ &= \lim_{x\longrightarrow 3}\frac{x-3}{(x+2)(x-3)\left(\sqrt{6+x}+3\right)}\\ &= \frac{1}{(3+2)\left(\sqrt{6+3}+3\right)}=\frac{1}{30}\end{aligned}$$

- $\lim\limits_{x\longrightarrow 1^+}\dfrac{x^2+x+1}{x^2-3x+2}$

  On a : $\lim\limits_{x\longrightarrow 1^+} x^2+x+1=3$. *Étudions le signe de* x^2-3x+2 :

  On a : $\Delta=1>0$ *donc l'équation* $x^2-3x+2=0$ *admet deux solutions réelles distinctes* 2 *et* 1. *Alors*

x	$-\infty$		1		2		$+\infty$
x^2-3x+2		$+$	0	$-$	0	$+$	

  par suite $\lim\limits_{x\longrightarrow 1^+} x^2-3x+2=0^-$, *donc par quotient :* $\lim\limits_{x\longrightarrow 1^+}\dfrac{x^2+x+1}{x^2-3x+2}=-\infty$.

- $\lim\limits_{x\longrightarrow 1^-}\dfrac{2x+3}{x^3-1}$

  On a : $\lim\limits_{x\longrightarrow 1^-} 2x+3=5$. *Étudions le signe de* x^3-1 :

  On a : $(\forall x\in\mathbb{R})$, $x^3-1=(x-1)\left(x^2+x+1\right)$, *et comme* $\Delta=-3<0$ *et* $a>0$ $(a=1)$ *alors* $(\forall x\in\mathbb{R})$, $x^2+x+1>0$.

  L'équation $x-1=0$ *admet unique solution* : 1. $(x-1=0 \iff x=1)$. *Donc*

x	$-\infty$		1		$+\infty$
$x-1$		$-$	0	$+$	
x^2+x+1		$+$		$+$	
x^3-1		$-$	0	$+$	

  par suite $\lim\limits_{x\longrightarrow 1^-} x^3-1=0^-$, *donc par quotient :* $\lim\limits_{x\longrightarrow 1^-}\dfrac{2x+3}{x^3-1}=-\infty$.

MÉTHODE 2

$\lim_{x\longrightarrow 1^-} x^3-1=0^-$ $\quad$ $(car\ :\ x<1 \Longrightarrow x^3<1^3 \Longrightarrow x^3-1<0)$, *donc par quotient :*

$\lim_{x\longrightarrow 1^-} \dfrac{2x+3}{x^3-1}=-\infty.$

EXERCICE 3 .

Calculons les limites suivantes :

- $\lim_{x\longrightarrow +\infty} \dfrac{x+\sqrt{x^2-4}}{x}=\text{"}\dfrac{+\infty}{+\infty}\text{"} \quad (F.I)$

  On a

$$\begin{aligned}\lim_{x\longrightarrow +\infty} \frac{x+\sqrt{x^2-4}}{x} &= \lim_{x\longrightarrow +\infty} \frac{x+\sqrt{x^2\left(1-\frac{4}{x^2}\right)}}{x}\\ &= \lim_{x\longrightarrow +\infty} \frac{x+x\sqrt{1-\frac{4}{x^2}}}{x}\\ &= \lim_{x\longrightarrow +\infty} \frac{x\left(1+\sqrt{1-\frac{4}{x^2}}\right)}{x}\\ &= \lim_{x\longrightarrow +\infty} 1+\sqrt{1-\frac{4}{x^2}}\end{aligned}$$

  et comme $\lim_{x\longrightarrow +\infty} \sqrt{1-\dfrac{4}{x^2}}=1$ *donc* $\lim_{x\longrightarrow +\infty} \dfrac{x+\sqrt{x^2-4}}{x}=2.$

- $\lim_{x\longrightarrow +\infty} \sqrt{x+\sqrt{x}}-\sqrt{x}=\text{"}+\infty-\infty\text{"} \quad (F.I)$

On a

$$\begin{aligned}
\lim_{x\longrightarrow+\infty}\sqrt{x+\sqrt{x}}-\sqrt{x} &= \lim_{x\longrightarrow+\infty}\frac{\left(\sqrt{x+\sqrt{x}}-\sqrt{x}\right)\left(\sqrt{x+\sqrt{x}}+\sqrt{x}\right)}{\left(\sqrt{x+\sqrt{x}}+\sqrt{x}\right)}\\
&= \lim_{x\longrightarrow+\infty}\frac{(x+\sqrt{x})-x}{\sqrt{x+\sqrt{x}}+\sqrt{x}}\\
&= \lim_{x\longrightarrow+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}}\\
&= \lim_{x\longrightarrow+\infty}\frac{\sqrt{x}}{\sqrt{x}\sqrt{1+\frac{\sqrt{x}}{x}}+\sqrt{x}}\\
&= \lim_{x\longrightarrow+\infty}\frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+\frac{1}{\sqrt{x}}}+1\right)}\\
&= \lim_{x\longrightarrow+\infty}\frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}+1}
\end{aligned}$$

et comme $\lim_{x\longrightarrow+\infty}\sqrt{1+\frac{1}{\sqrt{x}}}+1=2$ *donc* $\lim_{x\longrightarrow+\infty}\sqrt{x+\sqrt{x}}-\sqrt{x}=\frac{1}{2}$.

- $\lim_{x\longrightarrow-\infty}3x-1+\sqrt{9x^2+3x-2}="-\infty+\infty" \quad (F.I)$

  On a

$$\begin{aligned}
\lim_{x\longrightarrow-\infty}3x-1+\sqrt{9x^2+3x-2} &= \lim_{x\longrightarrow-\infty}\frac{\left(3x-1+\sqrt{9x^2+3x-2}\right)\left(3x-1-\sqrt{9x^2+3x-2}\right)}{3x-1-\sqrt{9x^2+3x-2}}\\
&= \lim_{x\longrightarrow-\infty}\frac{(3x-1)^2-(9x^2+3x-2)}{3x-1-\sqrt{9x^2+3x-2}}\\
&= \lim_{x\longrightarrow-\infty}\frac{9x^2-6x+1-9x^2-3x+2}{3x-1-\sqrt{9x^2+3x-2}}\\
&= \lim_{x\longrightarrow-\infty}\frac{-9x+3}{3x-1-\sqrt{9x^2+3x-2}}\\
&= \lim_{x\longrightarrow-\infty}\frac{x\left(-9+\frac{3}{x}\right)}{x\left(3-\frac{1}{x}+\sqrt{9+\frac{3}{x}-\frac{2}{x^2}}\right)}\\
&= \lim_{x\longrightarrow-\infty}\frac{-9+\frac{3}{x}}{3-\frac{1}{x}+\sqrt{9+\frac{3}{x}-\frac{2}{x^2}}}
\end{aligned}$$

et comme $\lim\limits_{x\longrightarrow -\infty} -9+\dfrac{3}{x}=-9$ *et* $\lim\limits_{x\longrightarrow -\infty} 3-\dfrac{1}{x}+\sqrt{9+\dfrac{3}{x}-\dfrac{2}{x^2}}=6$ *donc*

$$\lim_{x\longrightarrow -\infty} 3x-1+\sqrt{9x^2+3x-2}=\frac{-3}{2}$$

- $\lim\limits_{x\longrightarrow +\infty} x+2-\sqrt{x^2-x+6}=\text{"}+\infty-\infty\text{"}$ $(F.I)$

  On a

$$\begin{aligned}
\lim_{x\longrightarrow +\infty} x+2-\sqrt{x^2-x+6} &= \lim_{x\longrightarrow +\infty} \frac{\left(x+2-\sqrt{x^2-x+6}\right)\left(x+2+\sqrt{x^2-x+6}\right)}{x+2+\sqrt{x^2-x+6}} \\
&= \lim_{x\longrightarrow +\infty} \frac{(x+2)^2-(x^2-x+6)}{x+2+\sqrt{x^2-x+6}} \\
&= \lim_{x\longrightarrow +\infty} \frac{x^2+4x+4-x^2+x-6}{x+2+\sqrt{x^2-x+6}} \\
&= \lim_{x\longrightarrow +\infty} \frac{5x-2}{x+2+\sqrt{x^2-x+6}} \\
&= \lim_{x\longrightarrow +\infty} \frac{x\left(5-\dfrac{2}{x}\right)}{x\left(1+\dfrac{2}{x}+\sqrt{1-\dfrac{1}{x}+\dfrac{6}{x^2}}\right)} \\
&= \lim_{x\longrightarrow +\infty} \frac{5-\dfrac{2}{x}}{1+\dfrac{2}{x}+\sqrt{1-\dfrac{1}{x}+\dfrac{6}{x^2}}}
\end{aligned}$$

et comme $\lim\limits_{x\longrightarrow +\infty} 5-\dfrac{2}{x}=5$ *et* $\lim\limits_{x\longrightarrow +\infty} 1+\dfrac{2}{x}+\sqrt{1-\dfrac{1}{x}+\dfrac{6}{x^2}}=2$ *donc*

$$\lim_{x\longrightarrow +\infty} x+2-\sqrt{x^2-x+6}=\frac{5}{2}$$

EXERCICE 4 .

Calculons les limites suivantes :

- $\lim\limits_{x\longrightarrow \frac{\pi}{4}} \dfrac{2\cos x-\sqrt{2}}{x-\frac{\pi}{4}}=\text{"}\dfrac{0}{0}\text{"}$ $(F.I)$

  On utilise un changement de variable : On pose : $X=x-\dfrac{\pi}{4}$.

On a : $x \longrightarrow \frac{\pi}{4} \implies X \longrightarrow 0$ *donc*

$$\begin{aligned}
\lim_{x\longrightarrow\frac{\pi}{4}} \frac{2\cos x-\sqrt{2}}{x-\frac{\pi}{4}} &= \lim_{X\longrightarrow 0} \frac{2\cos\left(X+\frac{\pi}{4}\right)-\sqrt{2}}{X} \\
&= \lim_{X\longrightarrow 0} \frac{2\left(\cos X\cos\frac{\pi}{4}-\sin X\sin\frac{\pi}{4}\right)-\sqrt{2}}{X} \\
&= \lim_{X\longrightarrow 0} \frac{\sqrt{2}\cos X-\sqrt{2}\sin X-\sqrt{2}}{X} \\
&= \lim_{X\longrightarrow 0} \frac{-\sqrt{2}\,(1-\cos X)-\sqrt{2}\sin X}{X} \\
&= \lim_{X\longrightarrow 0} -\sqrt{2}\times\frac{1-\cos X}{X^2}\times X-\sqrt{2}\times\frac{\sin X}{X} \\
&= -\sqrt{2}\times\frac{1}{2}\times 0-\sqrt{2}\times 1=-\sqrt{2}
\end{aligned}$$

donc $\lim_{x\longrightarrow\frac{\pi}{4}} \frac{2\cos x-\sqrt{2}}{x-\frac{\pi}{4}}=-\sqrt{2}.$

- $\lim_{x\longrightarrow 1} \frac{\sin(\pi x)}{x-1} = \text{"}\frac{0}{0}\text{"}$ $(F.I)$

  On utilise un changement de variable : On pose : $X=x-1.$

  On a : $x \longrightarrow 1 \implies X \longrightarrow 0$ *donc*

$$\begin{aligned}
\lim_{x\longrightarrow 1} \frac{\sin(\pi x)}{x-1} &= \lim_{X\longrightarrow 0} \frac{\sin(\pi(X+1))}{X} \\
&= \lim_{X\longrightarrow 0} \frac{\sin(\pi X+\pi)}{X} \\
&= \lim_{X\longrightarrow 0} \frac{-\sin \pi X}{X} \qquad / \quad \sin(\pi+x)=-\sin x \\
&= \lim_{X\longrightarrow 0} -\frac{\sin \pi X}{\pi X}\times\pi \\
&= -\pi
\end{aligned}$$

comme : $\lim_{X\longrightarrow 0} \frac{\sin \pi X}{\pi X}=1$ *donc* $\lim_{x\longrightarrow 1} \frac{\sin(\pi x)}{x-1}=-\pi.$

- $\lim_{x\longrightarrow +\infty} \frac{x}{2+\cos x}$

  On a

$$-1\leq\cos x\leq 1 \iff 1\leq 2+\cos x\leq 3 \iff \frac{1}{3}\leq\frac{1}{2+\cos x}\leq 1$$

donc $\frac{1}{3}\leq\frac{1}{2+\cos x}\leq 1$ *et comme* $x\longrightarrow+\infty$ *alors* $x>0,$ *donc*

$$\frac{x}{3}\leq\frac{x}{2+\cos x}\leq x$$

et puisque $\lim\limits_{x\longrightarrow+\infty}\frac{x}{3}=+\infty$ *donc* $\lim\limits_{x\longrightarrow+\infty}\frac{x}{2+\cos x}=+\infty.$

- $\lim\limits_{x\longrightarrow 0}\frac{x\cos x+\sin x}{\sqrt{x+1}-1}=\text{"}\frac{0}{0}\text{"}\ (F.I)$

  On a

$$\begin{aligned}\lim_{x\longrightarrow 0}\frac{x\cos x+\sin x}{\sqrt{x+1}-1} &= \lim_{x\longrightarrow 0}\frac{x\left(\cos x+\frac{\sin x}{x}\right)\left(\sqrt{x+1}+1\right)}{\left(\sqrt{x+1}-1\right)\left(\sqrt{x+1}+1\right)}\\ &= \lim_{x\longrightarrow 0}\frac{x\left(\cos x+\frac{\sin x}{x}\right)\left(\sqrt{x+1}+1\right)}{x}\\ &= \lim_{x\longrightarrow 0}\left(\cos x+\frac{\sin x}{x}\right)\left(\sqrt{x+1}+1\right)\\ &= 4\end{aligned}$$

  donc $\lim\limits_{x\longrightarrow 0}\frac{x\cos x+\sin x}{\sqrt{x+1}-1}=1.$

EXERCICE 5 .

Soit $n\in\mathbb{N}^*$, *on considère la fonction numérique* f_n *définie par :*

$$f_n(x)=\frac{n-\sin x-\sin^2 x-\sin^3 x-...-\sin^n x}{1-\sin^2 x}$$

1. *On cherche* D_f :

   On a : $D_f=\left\{x\in\mathbb{R}/\ 1-\sin^2 x\neq 0\right\}$. *Résolvons dans* $\mathbb{R}$ *l'équation suivante :* $1-\sin^2 x=0$

   Soit $x\in\mathbb{R}$, *on a*

$$\begin{aligned}1-\sin^2 x &= 0 \iff (1+\sin x)(1-\sin x)=0\\ &\iff \sin x=1 \quad ou \quad \sin x=-1\\ &\iff x=\frac{\pi}{2}+2k\pi\ \ ou\ x=\frac{-\pi}{2}+2k\pi \quad /\ k\in\mathbb{Z}\end{aligned}$$

   donc :

$$\begin{aligned}D_f &= \left\{x\in\mathbb{R}/\ x\neq\frac{\pi}{2}+2k\pi \quad et \quad x\neq\frac{-\pi}{2}+2k\pi\ /\ k\in\mathbb{Z}\right\}\\ &= \mathbb{R}\backslash\left\{\frac{\pi}{2}+2k\pi,\frac{-\pi}{2}+2k\pi/\ k\in\mathbb{Z}\right\}\end{aligned}$$

2. *Monrons que :* $\lim\limits_{x\longrightarrow\frac{\pi}{2}}\frac{1-\sin^k x}{1-\sin^2 x}=\frac{k}{2}$

En utilisant l'identité remarquable suivante :

$$a^n - b^n = (a-b)\left(a^{n-1} + a^{n-2}b + ... + ab^{n-2} + b^{n-1}\right) = (a-b)\sum_{k=0}^{n-1} a^{n-1-k}b^k$$

Soit $k \in \{1, ..., n\}$, *on a*

$$\begin{aligned}
\lim_{x \longrightarrow \frac{\pi}{2}} \frac{1-\sin^k x}{1-\sin^2 x} &= \lim_{x \longrightarrow \frac{\pi}{2}} \frac{1^k - \sin^k x}{1-\sin^2 x} \\
& \lim_{x \longrightarrow \frac{\pi}{2}} \frac{(1-\sin x)\sum_{i=0}^{k-1} 1^{k-1-i}(\sin x)^i}{(1-\sin x)(1+\sin x)} \\
&= \lim_{x \longrightarrow \frac{\pi}{2}} \frac{\sum_{i=0}^{k-1}(\sin x)^i}{(1+\sin x)} \\
&= \frac{\sum_{i=0}^{k-1} 1^i}{2} \\
&= \frac{k-1-0+1}{2} \\
&= \frac{k}{2}
\end{aligned}$$

donc $\lim_{x \longrightarrow \frac{\pi}{2}} \frac{1-\sin^k x}{1-\sin^2 x} = \frac{k}{2}, \quad \forall k \in \{1, ..., n\}$.

3. Déduisons que : $\lim_{x \longrightarrow \frac{\pi}{2}} f_n(x) = \frac{n(n+1)}{4}$.

On a

$$
\begin{aligned}
\lim_{x\longrightarrow\frac{\pi}{2}} f_n(x) &= \lim_{x\longrightarrow\frac{\pi}{2}} \frac{n-\sin x-\sin^2 x-\sin^3 x-...-\sin^n x}{1-\sin^2 x} \\
&= \lim_{x\longrightarrow\frac{\pi}{2}} \frac{n-\displaystyle\sum_{k=1}^{n}\sin^k x}{1-\sin^2 x} \\
&= \lim_{x\longrightarrow\frac{\pi}{2}} \frac{n+\displaystyle\sum_{k=1}^{n}\left(1-\sin^k x-1\right)}{1-\sin^2 x} \\
&= \lim_{x\longrightarrow\frac{\pi}{2}} \frac{n+\displaystyle\sum_{k=1}^{n}\left(1-\sin^k x\right)-\sum_{k=1}^{n}1}{1-\sin^2 x} \\
&= \lim_{x\longrightarrow\frac{\pi}{2}} \frac{n+\displaystyle\sum_{k=1}^{n}\left(1-\sin^k x\right)-n}{1-\sin^2 x} \\
&= \lim_{x\longrightarrow\frac{\pi}{2}} \frac{\displaystyle\sum_{k=1}^{n}1-\sin^k x}{1-\sin^2 x} \\
&= \lim_{x\longrightarrow\frac{\pi}{2}} \sum_{k=1}^{n}\left(\frac{1-\sin^k x}{1-\sin^2 x}\right) \\
&= \lim_{x\longrightarrow\frac{\pi}{2}} \sum_{k=1}^{n}\frac{k}{2} \\
&= \frac{1}{2}\sum_{k=1}^{n}k \\
&= \frac{n\left(n+1\right)}{4} \quad / \quad \sum_{k=1}^{n}k=\frac{n\left(n+1\right)}{2}
\end{aligned}
$$

FIN

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