CORRECTION DEVOIR SURVEILLÉ

EXERCICE 1 .

Calculons les limites suivantes :

- $\lim\limits_{x\longrightarrow 0}\dfrac{x+\sin^2 x}{1-\cos x} = "\dfrac{0}{0}" \quad (F.I)$

  On a

$$\begin{aligned}\lim_{x\longrightarrow 0}\frac{x+\sin^2 x}{1-\cos x} &= \lim_{x\longrightarrow 0}\frac{x^2\left(\dfrac{1}{x}+\dfrac{\sin^2 x}{x^2}\right)}{\left(\dfrac{1-\cos^2 x}{x^2}\right)\times x^2}\\ &= \lim_{x\longrightarrow 0}\frac{\dfrac{1}{x}+\left(\dfrac{\sin x}{x}\right)^2}{\dfrac{1-\cos^2 x}{x^2}}\end{aligned}$$

et comme : $\lim\limits_{x\longrightarrow 0}\left(\dfrac{\sin x}{x}\right)^2 = 1$, $\lim\limits_{x\longrightarrow 0}\dfrac{1-\cos x}{x^2} = \dfrac{1}{2}$, *et :* $\begin{cases}\lim\limits_{x\longrightarrow 0^+}\dfrac{1}{x} = +\infty\\ \lim\limits_{x\longrightarrow 0^-}\dfrac{1}{x} = -\infty\end{cases}$ *alors :*
$\begin{cases}\lim\limits_{x\longrightarrow 0^+}\dfrac{x+\sin^2 x}{1-\cos x} = +\infty\\ \lim\limits_{x\longrightarrow 0^-}\dfrac{x+\sin^2 x}{1-\cos x} = -\infty\end{cases}$. *Or* $\lim\limits_{x\longrightarrow 0^+}\dfrac{x+\sin^2 x}{1-\cos x} \neq \lim\limits_{x\longrightarrow 0^-}\dfrac{x+\sin^2 x}{1-\cos x}$ *donc la fonction* $x \longmapsto \dfrac{x+\sin^2 x}{1-\cos x}$ *n'admet pas de limite au point* $x_0 = 0$.

- $\lim\limits_{x\longrightarrow \frac{\pi}{3}}\dfrac{\sin(3x)}{1-2\cos x} = "\dfrac{0}{0}" \quad (F.I)$

  On pose : $X = x - \dfrac{\pi}{3}$.

On a : $x \longrightarrow \dfrac{\pi}{3} \implies X \longrightarrow 0$ *donc*

$$\begin{aligned}
\lim_{x\longrightarrow\frac{\pi}{3}} \frac{\sin(3x)}{1-2\cos x} &= \lim_{X\longrightarrow 0} \frac{\sin\left(3\left(X+\frac{\pi}{3}\right)\right)}{1-2\cos\left(X+\frac{\pi}{3}\right)} \\
&= \lim_{X\longrightarrow 0} \frac{\sin(\pi+3X)}{1-2\cos\left(X+\frac{\pi}{3}\right)} \\
&= \lim_{X\longrightarrow 0} \frac{-\sin 3X}{1-2\left(\cos\frac{\pi}{3}\cos X-\sin\frac{\pi}{3}\sin X\right)} \\
&= \lim_{X\longrightarrow 0} \frac{-\sin(3X)}{1-2\left(\frac{1}{2}\cos X-\frac{\sqrt{3}}{2}\sin X\right)} \\
&= \lim_{X\longrightarrow 0} \frac{\sin(3X)}{1-\cos X+\sqrt{3}\sin X} \\
&= \lim_{X\longrightarrow 0} \frac{-\frac{\sin(3X)}{3X}\times 3X}{\frac{1-\cos X}{X^2}\times X^2+\sqrt{3}\frac{\sin X}{X}\times X} \\
&= \lim_{X\longrightarrow 0} \frac{-\frac{\sin(3X)}{3X}\times 3X}{X\left(\frac{1-\cos X}{X^2}\times X+\sqrt{3}\frac{\sin X}{X}\right)} \\
&= \lim_{X\longrightarrow 0} \frac{-3\times\frac{\sin(3X)}{3X}}{\frac{1-\cos X}{X^2}\times X+\sqrt{3}\frac{\sin X}{X}}
\end{aligned}$$

et comme $\lim\limits_{X\longrightarrow 0} \dfrac{1-\cos X}{X^2} = \dfrac{1}{2}$ *et* $\lim\limits_{X\longrightarrow 0} \dfrac{\sin X}{X} = 1$ *alors* $\lim\limits_{X\longrightarrow 0} \dfrac{-3\times\frac{\sin(3X)}{3X}}{\frac{1-\cos X}{X^2}\times X+\sqrt{3}\frac{\sin X}{X}} = -\sqrt{3}$ *donc* $\lim\limits_{x\longrightarrow\frac{\pi}{3}} \dfrac{\sin(3x)}{1-2\cos x} = -\sqrt{3}$.

- $\lim\limits_{x\longrightarrow\frac{\pi}{3}} \dfrac{\sqrt{3}\cos x-\sin x}{\sin 3x} =$ " $\dfrac{0}{0}$ "

  On pose : $X = x-\dfrac{\pi}{3}$.

$On\ a : x \longrightarrow \frac{\pi}{3} \Longrightarrow X \longrightarrow 0\ donc$

$$\begin{aligned}
\lim_{x\longrightarrow\frac{\pi}{3}} \frac{\sqrt{3}\cos - \sin x}{\sin 3x} &= \lim_{X\longrightarrow 0} \frac{\sqrt{3}\cos\left(X+\frac{\pi}{3}\right) - \sin\left(X+\frac{\pi}{3}\right)}{\sin\left(3\left(\frac{\pi}{3}+X\right)\right)} \\
&= \lim_{X\longrightarrow 0} \frac{\sqrt{3}\left(\cos\frac{\pi}{3}\cos X - \sin\frac{\pi}{3}\sin X\right) - \left(\sin\frac{\pi}{3}\cos X + \sin X\cos\frac{\pi}{3}\right)}{\sin(\pi+3X)} \\
&= \lim_{X\longrightarrow 0} \frac{\frac{\sqrt{3}}{2}\cos X - \frac{3}{2}\sin X - \frac{\sqrt{3}}{2}\cos X - \frac{\sin X}{2}}{-\sin 3X} \\
&= \lim_{X\longrightarrow 0} \frac{-2\sin X}{-\sin 3X} \\
&= \lim_{X\longrightarrow 0} \frac{-2\frac{\sin X}{X}\times X}{\frac{\sin 3X}{3X}\times 3X} \\
&= \lim_{X\longrightarrow 0} \frac{-2\frac{\sin X}{X}}{3\frac{\sin 3X}{3X}} = -\frac{2}{3}
\end{aligned}$$

$donc \lim_{x\longrightarrow\frac{\pi}{3}} \frac{\sqrt{3}\cos - \sin x}{\sin 3x} = -\frac{2}{3}.$

- $\lim_{x\longrightarrow -\frac{\pi}{2}} \frac{\cos x}{1+\sin x} = "\frac{0}{0}"\ (F.I)$

  $On\ pose : X = x + \frac{\pi}{2}.$

On a : $x \longrightarrow \dfrac{-\pi}{2} \Longrightarrow X \longrightarrow 0$ *donc*

$$\begin{aligned}
\lim_{x\longrightarrow -\frac{\pi}{2}} \frac{\cos x}{1+\sin x} &= \lim_{X\longrightarrow 0} \frac{\cos\left(X-\dfrac{\pi}{2}\right)}{1+\sin\left(X-\dfrac{\pi}{2}\right)} \\
&= \lim_{X\longrightarrow 0} \frac{\cos\left(\dfrac{\pi}{2}-X\right)}{1-\sin\left(\dfrac{\pi}{2}-X\right)} \\
&= \lim_{X\longrightarrow 0} \frac{\sin X}{1-\cos X} \\
&= \lim_{X\longrightarrow 0} \frac{\dfrac{\sin X}{X}\times X}{\dfrac{1-\cos X}{X^2}\times X^2} \\
&= \lim_{X\longrightarrow 0} \frac{\dfrac{\sin X}{X}}{\left(\dfrac{1-\cos X}{X^2}\right)\times X} \\
&= \lim_{X\longrightarrow 0} \frac{\dfrac{\sin X}{X}}{\left(\dfrac{1-\cos X}{X^2}\right)} \times \frac{1}{X}
\end{aligned}$$

et comme : $\displaystyle\lim_{X\longrightarrow 0} \frac{\sin X}{X} = 1$ *et* $\displaystyle\lim_{X\longrightarrow 0} \frac{1-\cos X}{X^2} = \frac{1}{2}$, *alors :* $\left\{\begin{array}{l} \displaystyle\lim_{X\longrightarrow 0^+} \frac{\dfrac{\sin X}{X}}{\left(\dfrac{1-\cos X}{X^2}\right)} \times \frac{1}{X} = +\infty \\ \\ \displaystyle\lim_{X\longrightarrow 0^-} \frac{\dfrac{\sin X}{X}}{\left(\dfrac{1-\cos X}{X^2}\right)} \times \frac{1}{X} = -\infty \end{array}\right.$

donc $\left\{\begin{array}{l} \displaystyle\lim_{X\longrightarrow -\frac{\pi}{2}^+} \frac{\cos x}{1+\sin x} = +\infty \\ \\ \displaystyle\lim_{X\longrightarrow \frac{-\pi}{2}^-} \frac{\cos x}{1+\sin x} = -\infty \end{array}\right.$ *Or* $\displaystyle\lim_{x\longrightarrow -\frac{\pi}{2}^+} \frac{\cos x}{1+\sin x} \neq \lim_{x\longrightarrow \frac{-\pi}{2}} \frac{\cos x}{1+\sin x}$ *donc la fonction* $x \longmapsto \dfrac{\cos x}{1+\sin x}$ *n'admet pas de limite au point* $x_0 = \dfrac{-\pi}{2}$.

EXERCICE 2 .

Discutons suivant les valeurs de m la limite $\displaystyle\lim_{x\longrightarrow +\infty} \sqrt{x^2-2x+5} - mx$:

- *Si* $m = 0$, *alors :* $\displaystyle\lim_{x\longrightarrow +\infty} \sqrt{x^2-2x+5} = +\infty$.

■ *Si* $m < 0$, *alors* $-m > 0$ *donc* $\lim_{x \longrightarrow +\infty} -mx = +\infty$ *et comme* $\lim_{x \longrightarrow +\infty} \sqrt{x^2-2x+5} = +\infty$ *d'où :* $\lim_{x \longrightarrow +\infty} \sqrt{x^2-2x+5} - mx = +\infty$.

■ *Si* $m > 0$, *alors on obtient :* $\lim_{x \longrightarrow +\infty} \sqrt{x^2-2x+5} - mx = " +\infty - \infty"$. (*F.I*)

Donc

$$\begin{aligned}\lim_{x \longrightarrow +\infty} \sqrt{x^2-2x+5} - mx &= \lim_{x \longrightarrow +\infty} x\sqrt{1-\frac{2}{x}+\frac{5}{x^2}} - mx \\ &= \lim_{x \longrightarrow +\infty} x\left(\sqrt{1-\frac{2}{x}+\frac{5}{x^2}} - m\right)\end{aligned}$$

et comme $\lim_{x \longrightarrow +\infty} x = +\infty$ *et* $\lim_{x \longrightarrow +\infty} \sqrt{1-\frac{2}{x}+\frac{5}{x^2}} - m = 1-m$, *on distingue* 3 *cas :*

- ■ *Si* $m > 1$, *alors* $\lim_{x \longrightarrow +\infty} x\left(\sqrt{1-\frac{2}{x}+\frac{5}{x^2}} - m\right) = -\infty$ *donc* $\lim_{x \longrightarrow +\infty} \sqrt{x^2-2x+5} - mx = -\infty$.
- ■ *Si* $m < 1$, *alors* $\lim_{x \longrightarrow +\infty} x\left(\sqrt{1-\frac{2}{x}+\frac{5}{x^2}} - m\right) = +\infty$ *donc* $\lim_{x \longrightarrow +\infty} \sqrt{x^2-2x+5} - mx = +\infty$.
- ■ *Si* $m = 1$, *alors*

$$\begin{aligned}\lim_{x \longrightarrow +\infty} x\left(\sqrt{1-\frac{2}{x}+\frac{5}{x^2}} - 1\right) &= \lim_{x \longrightarrow +\infty} x\left(\frac{-\frac{2}{x}+\frac{5}{x^2}}{\sqrt{1-\frac{2}{x}+\frac{5}{x^2}}+1}\right) \\ &= \lim_{x \longrightarrow +\infty} \frac{-2+\frac{5}{x}}{\sqrt{1-\frac{2}{x}+\frac{5}{x^2}}+1}\end{aligned}$$

et comme $\lim_{x \longrightarrow +\infty} -2+\frac{5}{x} = -2$ *et* $\lim_{x \longrightarrow +\infty} \sqrt{1-\frac{2}{x}+\frac{5}{x^2}}+1 = 2$ *donc* $\lim_{x \longrightarrow +\infty} \sqrt{x^2-2x+5} - mx = -1$.

donc on conclut que : $\lim_{x \longrightarrow +\infty} \sqrt{x^2-2x+5} - mx = \begin{cases} +\infty & , \ m < 1 \\ -1 , & m = 1 \\ -\infty , & m > 1 \end{cases}$.

EXERCICE 3 .

On considère la fonction f *définie par :* $\begin{cases} f(x) = \dfrac{\sqrt{x}-\sqrt{1+x^2}}{2+x} & ; \ si \ \ x \geq 0 \\ f(x) = \dfrac{\cos x - \sqrt{2+\sin x}}{x} & ; \ si \ \ x < 0 \end{cases}$

1. *Calculons* $\lim_{x\longrightarrow+\infty} f(x)$.

   On a

$$\begin{aligned}
\lim_{x\longrightarrow+\infty} f(x) &= \lim_{x\longrightarrow+\infty} \frac{\sqrt{x}-\sqrt{1+x^2}}{2+x} \\
&= \lim_{x\longrightarrow+\infty} \frac{\sqrt{x}-x\sqrt{1+\dfrac{1}{x^2}}}{2+x} \\
&= \lim_{x\longrightarrow+\infty} \frac{x\left(\dfrac{\sqrt{x}}{x}-\sqrt{1+\dfrac{1}{x^2}}\right)}{x\left(1+\dfrac{2}{x}\right)} \\
&= \lim_{x\longrightarrow+\infty} \frac{\dfrac{1}{\sqrt{x}}-\sqrt{1+\frac{1}{x^2}}}{1+\dfrac{2}{x}} \\
&= \lim_{x\longrightarrow+\infty} \frac{\dfrac{1}{\sqrt{x}}-\sqrt{1+\dfrac{1}{x^2}}}{1+\dfrac{2}{x}}
\end{aligned}$$

   et comme $\lim_{x\longrightarrow+\infty} 1+\dfrac{2}{x}=1$ *et* $\lim_{x\longrightarrow+\infty} \dfrac{1}{\sqrt{x}}-\sqrt{1+\dfrac{1}{x^2}}=-1$ *donc* $\lim_{x\longrightarrow+\infty} f(x)=-1$.

2. *Calculons* $\lim_{x\longrightarrow 0^+} f(x)$ *et* $\lim_{x\longrightarrow 0^-} f(x)$.

   On a : $\lim_{x\longrightarrow 0^+} f(x)=\lim_{x\longrightarrow 0^+} \dfrac{\sqrt{x}-\sqrt{1+x^2}}{2+x}=\dfrac{-1}{2}$ *et* $\lim_{x\longrightarrow 0^-} f(x)=\lim_{x\longrightarrow 0^-} \dfrac{\cos x-\sqrt{2+\sin x}}{x}=\dfrac{1-\sqrt{2}}{0^-}=+\infty$ $\left(1-\sqrt{2}<0\right)$. *Or* $\lim_{x\longrightarrow 0^+} f(x)\neq \lim_{x\longrightarrow 0^-} f(x)$ *donc la fonction f n'admet pas de limite au point* $x_0=0$.

3. **a)** *Montrons que :* $(\forall x\in\,]-\infty,0[)\,,\ |f(x)|\leq \dfrac{1+\sqrt{3}}{|x|}$.

   Soit $x\in\,]-\infty,0[$ *, on a :* $|f(x)|=\left|\dfrac{\cos x-\sqrt{2+\sin x}}{x}\right|=\dfrac{\left|\cos x-\sqrt{2+\sin x}\right|}{|x|}$.
   On a $-1\leq\sin x\leq 1$ *alors* $1\leq 2+\sin x\leq 3$ *donc :* $-\sqrt{3}\leq-\sqrt{2+\sin x}\leq-1$ $(\clubsuit)$
   et comme $-1\leq\cos x\leq 1$ *et d'après* $(\clubsuit)$ *on obtient*

$$1-\sqrt{3}\leq\cos x-\sqrt{2+\sin x}\leq 0$$

   et puisque $0\leq 1+\sqrt{3}$ *alors*

$$-1-\sqrt{3}\leq\cos x-\sqrt{2+\sin x}\leq 1+\sqrt{3}$$

donc : $\left|\cos x-\sqrt{2+\sin x}\right|\leq 1+\sqrt{3}$, *d'où* $\dfrac{\left|\cos x-\sqrt{2+\sin x}\right|}{|x|}\leq\dfrac{1+\sqrt{3}}{|x|}$ *ceci signifie que*

$$(\forall x\in\,]-\infty,0[)\,,\quad |f(x)|\leq\frac{1+\sqrt{3}}{|x|}$$

b) *Déduisons* $\lim\limits_{x\longrightarrow-\infty}f(x)$:

Soit $x\in\,]-\infty,0[$, *on a* $|f(x)|\leq\dfrac{1+\sqrt{3}}{|x|}$ *c'est-à-dire* $|f(x)-0|\leq\dfrac{1+\sqrt{3}}{|x|}$ *et comme* $\lim\limits_{x\longrightarrow-\infty}\dfrac{1+\sqrt{3}}{|x|}=0$ *donc* $\lim\limits_{x\longrightarrow-\infty}f(x)=0$.

EXERCICE 4 .

Soit f *la fonction numérique définie par :* $\begin{cases} f(x)=\dfrac{\sqrt{x}-1}{2-\sqrt{3+x}} & si\quad x>1\\[2ex] f(x)=\dfrac{\sqrt{1-x}}{2x^2+x-3} & si\quad x<1\end{cases}$

1. *On cherche* D_f.

   On pose : $\begin{cases} f_1(x)=\dfrac{\sqrt{x}-1}{2-\sqrt{3+x}} & et\;\; I=\,]1,+\infty[\\[2ex] f_2(x)=\dfrac{\sqrt{1-x}}{2x^2+x-3} & et\;\; J=\,]-\infty,1[\end{cases}$

   On a d'une part

$$\begin{aligned} D_{f_1} &= \left\{x\in\mathbb{R}/\ x\geq 0\ \ et\ 3+x\geq 0\ \ et\ 2-\sqrt{3+x}\neq 0\right\}\\ &= \left\{x\in\mathbb{R}/\ x\geq 0\ \ et\ x\geq -3\ \ et\ \sqrt{3+x}\neq 2\right\}\\ &= \left\{x\in\mathbb{R}/\ x\geq 0\ \ et\ \sqrt{3+x}\neq 2\right\}\end{aligned}$$

   Résolvons dans $[0,+\infty[$ *l'équation :* $\sqrt{3+x}=2$
   On a : $\sqrt{3+x}=2\iff 3+x=4\iff x=1$ *donc* $D_{f_1}=[0,1[\cup]1,+\infty[$, *par suite* :

$$D_{f_1}\cap I=\,]1,+\infty[$$

   D'autre part, on a

$$\begin{aligned} D_{f_2} &= \left\{x\in\mathbb{R}/\ 1-x\geq 0\ \ et\ 2x^2+x-3\neq 0\right\}\\ &= \left\{x\in\mathbb{R}/\ x\leq 1\ \ et\ 2x^2+x-3\neq 0\right\}\end{aligned}$$

L'équation $2x^2+x-3=0$ *admet deux solutions réelles distinctes :* $\dfrac{-3}{2}$ *et* 1*, d'où*

$$\begin{aligned} D_{f_2} &= \left\{x\in\mathbb{R}/\ x\leq 1 \text{ et } x\neq 1 \text{ et } x\neq \frac{-3}{2}\right\} \\ &= \left]-\infty, \frac{-3}{2}\right[\cup \left]\frac{-3}{2}, 1\right[\end{aligned}$$

par suite : $D_{f_2}\cap J = \left]-\infty, \dfrac{-3}{2}\right[\cup \left]\dfrac{-3}{2}, 1\right[$. *Finalement :*

$$\begin{aligned} D_f &= (D_{f_1}\cap I)\cup(D_{f_2}\cap J) \\ &= \left]-\infty, \frac{-3}{2}\right[\cup \left]\frac{-3}{2}, 1\right[\cup]1,+\infty[\end{aligned}$$

2. *Calculons :* $\lim\limits_{x\longrightarrow+\infty} f(x)$ *et* $\lim\limits_{x\longrightarrow-\infty} f(x)$.

- *On a*

$$\begin{aligned} \lim_{x\longrightarrow+\infty} f(x) &= \lim_{x\longrightarrow+\infty} \frac{\sqrt{x}-1}{2-\sqrt{3+x}} \\ &= \lim_{x\longrightarrow+\infty} \frac{\sqrt{x}-1}{2-\sqrt{x}\left(\sqrt{1+\frac{3}{x}}\right)} \\ &= \lim_{x\longrightarrow+\infty} \frac{\sqrt{x}\left(1-\frac{1}{\sqrt{x}}\right)}{\sqrt{x}\left(\frac{2}{\sqrt{x}}-\sqrt{1+\frac{3}{x}}\right)} \\ &= \lim_{x\longrightarrow+\infty} \frac{1-\frac{1}{\sqrt{x}}}{\frac{2}{\sqrt{x}}-\sqrt{1+\frac{3}{x}}} \end{aligned}$$

et comme $\lim\limits_{x\longrightarrow+\infty} 1-\dfrac{1}{\sqrt{x}} = 1$ *et* $\lim\limits_{x\longrightarrow+\infty} \dfrac{2}{\sqrt{x}}-\sqrt{1+\dfrac{3}{x}} = -1$ *donc* $\lim\limits_{x\longrightarrow+\infty} f(x) = -1$.

- *On a*

$$\begin{aligned} \lim_{x\longrightarrow-\infty} f(x) &= \lim_{x\longrightarrow-\infty} \frac{\sqrt{1-x}}{2x^2+x-3} \\ &= \lim_{x\longrightarrow-\infty} \frac{(1-x)}{(2x^2+x-3)\sqrt{1-x}} \\ &= \lim_{x\longrightarrow-\infty} \frac{1-x}{2x^2+x-3}\times\frac{1}{\sqrt{1-x}} \end{aligned}$$

et comme $\lim\limits_{x\longrightarrow -\infty}\dfrac{1-x}{2x^2+x-3}=\lim\limits_{x\longrightarrow -\infty}\dfrac{-x}{2x^2}=\lim\limits_{x\longrightarrow -\infty}\dfrac{-1}{2x}=0$ *et* $\lim\limits_{x\longrightarrow -\infty}\dfrac{1}{\sqrt{1-x}}=0$ *donc* $\lim\limits_{x\longrightarrow -\infty}f(x)=0$.

3. *Calculons :* $\lim\limits_{x\longrightarrow 1^+}f(x)$ *et* $\lim\limits_{x\longrightarrow 1^-}f(x)$.

- $\lim\limits_{x\longrightarrow 1^+}f(x)=\lim\limits_{x\longrightarrow 1^+}\dfrac{\sqrt{x}-1}{2-\sqrt{3+x}}=\text{"}\dfrac{0}{0}\text{"}$ *(F.I)*
  On a

$$
\begin{aligned}
\lim_{x\longrightarrow 1^+}f(x) &= \lim_{x\longrightarrow 1^+}\frac{\sqrt{x}-1}{2-\sqrt{3+x}}\\
&= \lim_{x\longrightarrow 1^+}\frac{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)\left(2+\sqrt{3+x}\right)}{\left(2-\sqrt{3+x}\right)\left(2+\sqrt{3+x}\right)\left(\sqrt{x}+1\right)}\\
&= \lim_{x\longrightarrow 1^+}\frac{(x-1)\left(2+\sqrt{3+x}\right)}{(4-3-x)\left(\sqrt{x}+1\right)}\\
&= \lim_{x\longrightarrow 1^+}\frac{(x-1)\left(2+\sqrt{3+x}\right)}{(1-x)\left(\sqrt{x}+1\right)}\\
&= \lim_{x\longrightarrow 1^+}\frac{(x-1)\left(2+\sqrt{3+x}\right)}{-(x-1)\left(\sqrt{x}+1\right)}\\
&= \lim_{x\longrightarrow 1^+}-\frac{\left(2+\sqrt{3+x}\right)}{\left(\sqrt{x}+1\right)}=-2
\end{aligned}
$$

  donc $\lim\limits_{x\longrightarrow 1^+}f(x)=-2$.

- $\lim\limits_{x\longrightarrow 1^-}f(x)=\lim\limits_{x\longrightarrow 1^-}\dfrac{\sqrt{1-x}}{2x^2+x-3}=\text{"}\dfrac{0}{0}\text{"}$ *(F.I)*
  On a

$$
\begin{aligned}
\lim_{x\longrightarrow 1^-}f(x) &= \lim_{x\longrightarrow 1^-}\frac{\sqrt{1-x}}{2x^2+x-3}\\
&= \lim_{x\longrightarrow 1^-}\frac{(1-x)}{(2x+3)(x-1)\sqrt{1-x}}\\
&= \lim_{x\longrightarrow 1^-}\frac{-(x-1)}{(2x+3)(x-1)\sqrt{1-x}}\\
&= \lim_{x\longrightarrow 1^-}\frac{-1}{(2x+3)\sqrt{1-x}}\\
&= \lim_{x\longrightarrow 1^-}\frac{-1}{(2x+3)}\times\frac{1}{\sqrt{1-x}}
\end{aligned}
$$

  et comme $\lim\limits_{x\longrightarrow 1^-}\dfrac{-1}{(2x+3)}=\dfrac{-1}{5}$ *et* $\lim\limits_{x\longrightarrow 1^-}\dfrac{1}{\sqrt{1-x}}=+\infty$ *donc* $\lim\limits_{x\longrightarrow 1^-}f(x)=-\infty$.
  Puisque $\lim\limits_{x\longrightarrow 1^+}f(x)\neq\lim\limits_{x\longrightarrow 1^-}f(x)$ *donc la fonction* f *n'admet pas de limite au point* $x_0=1$.

4. Étudions la limite de la fonction f au point $x_1 = \frac{-3}{2}$.

On a $\lim\limits_{x \longrightarrow \frac{-3}{2}} \sqrt{1-x} = \sqrt{\frac{5}{2}}$ *et* $\lim\limits_{x \longrightarrow \frac{-3}{2}} 2x^2 + x - 3 = 0$.*Étudie le signe de l'expression* $2x^2 + x - 3$.

L'équation $2x^2 + x - 3 = 0$ *admet deux solutions réelles distinctes* 1 *et* $\frac{-3}{2}$, *alors*

x	$-\infty$		$-3/2$		1		$+\infty$
$2x^2+x-3$		$+$	0	$-$	0	$+$	

donc $\lim\limits_{x \longrightarrow \frac{-3}{2}^-} \frac{\sqrt{1-x}}{2x^2+x-3} = +\infty$ *et* $\lim\limits_{x \longrightarrow \frac{-3}{2}^+} \frac{\sqrt{1-x}}{2x^2+x-3} = -\infty$, *et comme* $\lim\limits_{x \longrightarrow \frac{-3}{2}^-} \frac{\sqrt{1-x}}{2x^2+x-3} \neq \lim\limits_{x \longrightarrow \frac{-3}{2}^+} \frac{\sqrt{1-x}}{2x^2+x-3}$ *donc la fonction donc la fonction* $x \longmapsto \frac{\sqrt{1-x}}{2x^2+x-3}$ *n'admet pas de limite au point* $x_1 = \frac{-3}{2}$.

FIN

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