Correction de la série N1 sur la fonction Arctangente

Exercice 1 .

1. *Simplifions les expressions suivantes* :

■ $A = \arctan 2 + \arctan\left(\frac{1}{2}\right)$

On a pour tout $x \in \left]0, +\infty\right[$, $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$.

En particulier pour $x = 2$, *on obtient* :

$$A = \arctan 2 + \arctan\left(\frac{1}{2}\right) = \frac{\pi}{2}.$$

■ *L'expression* : $B = \arctan\left(1 - \sqrt{2}\right) - \arctan\left(1 + \sqrt{2}\right)$.

On sait que : $1 - \sqrt{2} = \tan\left(\frac{-\pi}{8}\right)$, *alors* : $\arctan\left(1 - \sqrt{2}\right) = \arctan\left(\tan\left(\frac{-\pi}{8}\right)\right) = -\frac{\pi}{8}$.

Donc

$$\begin{aligned}
B &= -\frac{\pi}{8} - \arctan\left(1 + \sqrt{2}\right) \\
&= -\frac{\pi}{4} - \frac{\pi}{8} + \frac{\pi}{4} - \arctan\left(1 + \sqrt{2}\right) \\
&= \frac{-3\pi}{8} + \left(\arctan(1) - \arctan\left(1 + \sqrt{2}\right)\right) \\
&= \frac{-3\pi}{8} + \arctan\left(\frac{1 - 1 - \sqrt{2}}{1 + 1 + \sqrt{2}}\right) \quad / \ \arctan a - \arctan b = \arctan\left(\frac{a-b}{1+ab}\right) \\
\textit{avec } a &\geq 0 \textit{ et } b \geq 0 \\
&= \frac{-3\pi}{8} + \arctan\left(\frac{2 - 2\sqrt{2}}{2}\right) \\
&= \frac{-3\pi}{8} + \arctan\left(1 - \sqrt{2}\right) \\
&= \frac{-3\pi}{8} - \frac{\pi}{8} \\
&= \frac{-4\pi}{8} \\
&= -\frac{\pi}{2}
\end{aligned}$$

2. Montrons les égalités suivantes :

■ $\arctan\frac{1}{2}+\arctan\frac{1}{3}=\frac{\pi}{4}$.

On a : $0<\frac{1}{2}<1$ *et* $0<\frac{1}{3}<1$, *alors* $\arctan\frac{1}{2}\in\left]0,\frac{\pi}{4}\right[$ *et* $\arctan\frac{1}{3}\in\left]0,\frac{\pi}{4}\right[$, *d'où*

$$\arctan\frac{1}{2}+\arctan\frac{1}{3}\in\left]0,\frac{\pi}{2}\right[.$$

Donc

$$\begin{aligned}\tan\left(\arctan\frac{1}{2}+\arctan\frac{1}{3}\right) &= \frac{\tan\left(\arctan\frac{1}{2}\right)+\tan\left(\arctan\frac{1}{3}\right)}{1-\tan\left(\arctan\frac{1}{2}\right).\tan\left(\arctan\frac{1}{3}\right)}\\ &= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\\ &= 1\end{aligned}$$

d'où

$$\tan\left(\arctan\frac{1}{2}+\arctan\frac{1}{3}\right)=1 \qquad (\clubsuit)$$

on en déduit que :

$$\begin{aligned}(\clubsuit) &\iff \arctan\left(\tan\left(\arctan\frac{1}{2}+\arctan\frac{1}{3}\right)\right)=\arctan 1\\ &\iff \arctan\frac{1}{2}+\arctan\frac{1}{3}=\frac{\pi}{4}.\end{aligned}$$

d'où on obtient

$$\arctan\frac{1}{2}+\arctan\frac{1}{3}=\frac{\pi}{4}$$

■ $\arctan\frac{1}{2}+\arctan\frac{1}{5}+\arctan\frac{1}{8}=\frac{\pi}{4}$.

On a $0<\frac{1}{5}<1$ *et* $0<\frac{1}{8}<1$, *alors* $\arctan\frac{1}{5}\in\left]0,\frac{\pi}{4}\right[$ *et* $\arctan\frac{1}{8}\in\left]0,\frac{\pi}{4}\right[$, *d'où*

$$\left(\arctan\frac{1}{5}+\arctan\frac{1}{8}\right)\in\left]0,\frac{\pi}{2}\right[.$$

Donc

$$\begin{aligned}
\tan\left(\arctan\frac{1}{5}+\arctan\frac{1}{8}\right) &= \frac{\tan\left(\arctan\frac{1}{5}\right)+\tan\left(\arctan\frac{1}{8}\right)}{1-\tan\left(\arctan\frac{1}{5}\right).\tan\left(\arctan\frac{1}{8}\right)}\\
&= \frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}}\\
&= \frac{1}{3}
\end{aligned}$$

d'où

$$\tan\left(\arctan\frac{1}{5}+\arctan\frac{1}{8}\right)=\frac{1}{3} \qquad (\clubsuit)$$

on en déduit que :

$$\begin{aligned}
(\clubsuit) &\iff \arctan\left(\tan\left(\arctan\frac{1}{5}+\arctan\frac{1}{8}\right)\right)=\arctan\frac{1}{3}\\
&\iff \arctan\frac{1}{5}+\arctan\frac{1}{8}=\arctan\frac{1}{3}.
\end{aligned}$$

D'autre part, on a : $\arctan\frac{1}{3}\in\left]0,\frac{\pi}{4}\right[$ *et* $\arctan\frac{1}{2}\in\left]0,\frac{\pi}{4}\right[$,*alors*

$$\left(\arctan\frac{1}{3}+\arctan\frac{1}{2}\right)\in\left]0,\frac{\pi}{2}\right[.$$

Donc

$$\begin{aligned}
\tan\left(\arctan\frac{1}{3}+\arctan\frac{1}{2}\right) &= \frac{\tan\left(\arctan\frac{1}{3}\right)+\tan\left(\arctan\frac{1}{2}\right)}{1-\tan\left(\arctan\frac{1}{3}\right).\tan\left(\arctan\frac{1}{2}\right)}\\
&= \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\\
&= 1
\end{aligned}$$

d'où

$$\tan\left(\arctan\frac{1}{3}+\arctan\frac{1}{2}\right)=1 \qquad (\clubsuit\clubsuit)$$

on en déduit que :

$$\begin{aligned}
(\clubsuit\clubsuit) &\iff \arctan\left(\tan\left(\arctan\frac{1}{3}+\arctan\frac{1}{2}\right)\right)=\arctan 1\\
&\iff \arctan\frac{1}{3}+\arctan\frac{1}{2}=\frac{\pi}{4}
\end{aligned}$$

puisuqe $\arctan\dfrac{1}{3}=\arctan\dfrac{1}{5}+\arctan\dfrac{1}{8}$, *donc*

$$\arctan\frac{1}{5}+\arctan\frac{1}{8}+\arctan\frac{1}{2}=\frac{\pi}{4}$$

$4\arctan\dfrac{1}{5}-\arctan\dfrac{1}{239}=\dfrac{\pi}{4}$.

On a $\arctan\dfrac{1}{5}\in\left]0,\dfrac{\pi}{8}\right[$, *alors* $4\arctan\dfrac{1}{5}\in\left]0,\dfrac{\pi}{4}\right[$.

Et comme

$$\tan\left(2\arctan\frac{1}{5}\right)=\frac{2\tan\left(\arctan\frac{1}{5}\right)}{1-\tan^2\left(\arctan\frac{1}{5}\right)}=\frac{\frac{2}{5}}{1-\frac{1}{25}}=\frac{5}{12},$$

puis

$$\tan\left(4\arctan\frac{1}{5}\right)=\frac{2\tan\left(2\arctan\frac{1}{5}\right)}{1-\tan^2\left(2\arctan\frac{1}{5}\right)}=\frac{\frac{5}{6}}{1-\left(\frac{5}{12}\right)^2}=\frac{120}{119}.$$

d'où

$$\tan\left(4\arctan\frac{1}{5}\right)=\frac{120}{119}\qquad(\diamondsuit)$$

on en déduit que :

$$\begin{aligned}(\diamondsuit)\quad&\Longleftrightarrow\quad\arctan\left(\tan\left(4\arctan\frac{1}{5}\right)\right)=\arctan\frac{120}{119}\\&\Longleftrightarrow\quad 4\arctan\frac{1}{5}=\arctan\frac{120}{119}.\end{aligned}$$

D'autre part, on a : $\dfrac{120}{119}\in\left]0,\sqrt{3}\right[$, *alors* $\arctan\dfrac{120}{119}\in\left]0,\dfrac{\pi}{3}\right[$ *et* $\arctan\dfrac{1}{239}\in\left]0,\dfrac{\pi}{4}\right[$,*alors*

$$\left(\arctan\frac{120}{119}-\arctan\frac{1}{239}\right)\in\left]\frac{-\pi}{4},\frac{\pi}{3}\right[.$$

Donc

$$\begin{aligned}\tan\left(\arctan\frac{120}{119}-\arctan\frac{1}{239}\right)&=\frac{\tan\left(\arctan\frac{120}{119}\right)-\tan\left(\arctan\frac{1}{239}\right)}{1+\tan\left(\arctan\frac{120}{119}\right).\tan\left(\arctan\frac{1}{239}\right)}\\&=\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\times\frac{1}{239}}\\&=1\end{aligned}$$

d'où

$$\tan\left(\arctan\frac{120}{119}-\arctan\frac{1}{239}\right)=1 \quad (\diamondsuit\diamondsuit)$$

on en déduit que :

$$\begin{aligned}(\diamondsuit\diamondsuit) \quad &\Longleftrightarrow \quad \arctan\left(\tan\left(\arctan\frac{120}{119}-\arctan\frac{1}{239}\right)\right)=\arctan 1\\ &\Longleftrightarrow \quad \arctan\frac{120}{119}-\arctan\frac{1}{239}=\frac{\pi}{4}\end{aligned}$$

puisuqe $\arctan\frac{120}{119}=4\arctan\frac{1}{5}$, *donc*

$$4\arctan\frac{1}{5}-\arctan\frac{1}{239}=\frac{\pi}{4}$$

3. *On pose :* $\alpha=2\arctan\frac{1}{2}-\arctan\frac{1}{7}$.

a) *Vérifions que :* $0<\alpha<\frac{\pi}{3}$.

On a : $\frac{1}{2}\in\left]0,\frac{\sqrt{3}}{3}\right[$ *et* $\frac{1}{7}\in\left]0,1\right[$, *alors* $2\arctan\frac{1}{2}\in\left]0,\frac{\pi}{3}\right[$ *et* $\arctan\frac{1}{7}\in\left]0,\frac{\pi}{4}\right[$, *donc*

$$\left(2\arctan\frac{1}{2}-\arctan\frac{1}{7}\right)\in\left]\frac{-\pi}{4},\frac{\pi}{3}\right[$$

C'est-à-dire $\alpha\in\left]\frac{-\pi}{4},\frac{\pi}{3}\right[$ *donc* $\alpha<\frac{\pi}{3}$. (1)

D'autre part, on a

$$\begin{aligned}\frac{1}{2} \quad > \quad \frac{1}{7} &\Longleftrightarrow \arctan\frac{1}{2}>\arctan\frac{1}{7}\\ &\Longrightarrow 2\arctan\frac{1}{2}>2\arctan\frac{1}{7}>\arctan\frac{1}{7}\\ &\Longrightarrow 2\arctan\frac{1}{2}>\arctan\frac{1}{7}\\ &\Longrightarrow 2\arctan\frac{1}{2}-\arctan\frac{1}{7}>0\\ &\Longrightarrow \alpha>0.\end{aligned}$$

Ceci signifie que $\alpha>0$. (2)

D'où d'après (1) *et* (2) *on déduit que*

$$0<\alpha<\frac{\pi}{3}$$

b) ■ *Calculons* $\tan\alpha$.

On a : $\alpha \in \left]0, \frac{\pi}{3}\right[$. *Donc*

$$\begin{aligned}
\tan\alpha &= \tan\left(2\arctan\frac{1}{2} - \arctan\frac{1}{7}\right) \\
&= \frac{\tan\left(2\arctan\frac{1}{2}\right) - \tan\left(\arctan\frac{1}{7}\right)}{1+\tan\left(2\arctan\frac{1}{2}\right).\tan\left(\arctan\frac{1}{7}\right)}
\end{aligned}$$

Calculons $\tan\left(2\arctan\frac{1}{2}\right)$.

On a : $2\arctan\frac{1}{2} \in \left]0, \frac{\pi}{3}\right[$, *donc*

$$\tan\left(2\arctan\frac{1}{2}\right) = \frac{2\tan\left(\arctan\frac{1}{2}\right)}{1-\tan^2\left(\arctan\frac{1}{2}\right)} = \frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} = \frac{4}{3},$$

d'où

$$\begin{aligned}
\tan\alpha &= \frac{\tan\left(2\arctan\frac{1}{2}\right) - \tan\left(\arctan\frac{1}{7}\right)}{1+\tan\left(2\arctan\frac{1}{2}\right).\tan\left(\arctan\frac{1}{7}\right)} \\
&= \frac{\frac{4}{3}-\frac{1}{7}}{1+\frac{4}{3}\times\frac{1}{7}} \\
&= 1
\end{aligned}$$

d'où

$$\tan(\alpha) = 1 \quad (*)$$

on en déduit que :

$$\begin{aligned}
(*) &\iff \arctan(\tan(\alpha)) = \arctan 1 \\
&\iff \alpha = \arctan 1 \\
&\iff \alpha = \arctan\left(\tan\frac{\pi}{4}\right) \\
&\iff \alpha = \frac{\pi}{4}
\end{aligned}$$

Exercice 2 *On résout les équations suivantes dans* $\mathbb{R}$.

1. *L'équation* $(E_1) : \arctan 3x = \dfrac{\pi}{8}$ *est définie sur* $\mathbb{R}$.

   Soit $x \in \mathbb{R}$.

$$\begin{aligned}(E_1) \quad &\Longleftrightarrow \quad \tan(\arctan 3x) = \tan\frac{\pi}{8} \\ &\Longleftrightarrow \quad 3x = \tan\frac{\pi}{8} \\ &\Longleftrightarrow \quad x = \frac{\tan\frac{\pi}{8}}{3}.\end{aligned}$$

   Donc l'ensemble des solutions de l'équation est :

$$S = \left\{ \frac{\tan\left(\frac{\pi}{8}\right)}{3} \right\}.$$

2. *L'équation* $(E_2) : \arctan(x^2 - x) = \dfrac{3\pi}{4}$ *est définie sur* $\mathbb{R}$.

   Soit $x \in \mathbb{R}$.

$$\begin{aligned}(E_2) \quad &\Longleftrightarrow \quad \tan\left(\arctan\left(x^2 - x\right)\right) = \tan\frac{3\pi}{4} \\ &\Longleftrightarrow \quad x^2 - x = \tan\left(\pi - \frac{\pi}{4}\right) \\ &\Longleftrightarrow \quad x^2 - x = \tan\left(-\frac{\pi}{4}\right) \\ &\Longleftrightarrow \quad x^2 - x = -1 \\ &\Longleftrightarrow \quad x^2 - x + 1 = 0\end{aligned}$$

   Comme l'équation $(E') : x^2 - x + 1 = 0$ *a pour discriminant négatif, ceci signifie qu'elle n'admet aucune solution réelle. Par suite l'équation* (E_2) *n'admet aucune solution dans* $\mathbb{R}$. *C'est-à-dire*

$$S = \varnothing.$$

3. *L'équation* $(E_3) : \arctan\sqrt{x} = -\dfrac{\pi}{4}$ *est définie sur* $\mathbb{R}^+$.

   Or $\sqrt{x} \in \mathbb{R}^+$ *donc* $\arctan\sqrt{x} \in \left[0, \dfrac{\pi}{2}\right[$, *comme* $\dfrac{-\pi}{4} \notin \left[0, \dfrac{\pi}{2}\right[$. *Donc*

$$S = \varnothing.$$

4. *L'équation* $(E_4) : \arctan(x) + \arctan(2x) = \dfrac{\pi}{3}$ *est définie sur* $\mathbb{R}$.

   On sait que le signe de $\arctan x$ *est celui de* x, *et puisque* $\dfrac{\pi}{3} > 0$ *alors nécessairement* $x > 0$.

   Soit $x \in \left]0, +\infty\right[$.

On a

$$0 < \arctan x < \frac{\pi}{2} \quad et \quad \frac{-\pi}{6} < \frac{\pi}{3} - \arctan x < \frac{\pi}{3}$$

donc

$$\begin{aligned}
(E_4) \quad &\Longleftrightarrow \quad \arctan 2x = \frac{\pi}{3} - \arctan x \\
&\Longleftrightarrow \quad 2x = \tan\left(\frac{\pi}{3} - \arctan x\right) \\
&\Longleftrightarrow \quad 2x = \frac{\tan\frac{\pi}{3} - \tan(\arctan x)}{1 + \tan\frac{\pi}{3}.\tan(\arctan(x))} \\
&\Longleftrightarrow \quad 2x = \frac{\sqrt{3} - x}{1 + \sqrt{3}.x} \\
&\Longleftrightarrow \quad 2x + 2\sqrt{3}x^2 = \sqrt{3} - x \\
&\Longleftrightarrow \quad 2\sqrt{3}x^2 + 3x - \sqrt{3} = 0
\end{aligned}$$

Comme les solutions de l'équation $(E') : 2\sqrt{3}x^2 + 3x - \sqrt{3} = 0$ *sont :* $\frac{1}{4}\sqrt{11} - \frac{1}{4}\sqrt{3}$ *et* $\frac{-\sqrt{99}}{12} - \frac{1}{4}\sqrt{3}$. *On ne garde que la solution positive.*
Donc l'ensemble des solutions de l'équation (E_4) *est :*

$$S = \left\{\frac{1}{4}\sqrt{11} - \frac{1}{4}\sqrt{3}\right\}$$

5. *On résout dans* $\mathbb{R}$ *l'équation :* $(E_5) : \arctan x = \arctan\frac{1}{2} + \arctan\frac{1}{3}$.
   Soit $x \in \mathbb{R}$.
   On a : $0 < \frac{1}{2} < \frac{\sqrt{3}}{3}$ *et* $0 < \frac{1}{3} < \frac{\sqrt{3}}{3}$, *alors* $0 < \arctan\frac{1}{2} < \frac{\pi}{6}$ *et* $0 < \arctan\frac{1}{3} < \frac{\pi}{6}$.
   Donc

$$0 < \arctan\frac{1}{2} + \arctan\frac{1}{3} < \frac{\pi}{3}$$

Par suite

$$\begin{aligned}
(E) \quad &\Longleftrightarrow \quad \tan(\arctan(x)) = \tan\left(\arctan\frac{1}{2} + \arctan\frac{1}{3}\right) \\
&\Longleftrightarrow \quad x = \frac{\tan\left(\arctan\frac{1}{2}\right) + \tan\left(\arctan\frac{1}{3}\right)}{1 - \tan\left(\arctan\frac{1}{2}\right)\tan\left(\arctan\frac{1}{3}\right)} \\
&\Longleftrightarrow \quad x = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} \\
&\Longleftrightarrow \quad x = 1.
\end{aligned}$$

Donc l'ensemble des solutions de l'équation est : $S = \{1\}$.

6. *L'équation* (E_6) *est définie sur* $\mathbb{R}$.

   On sait que le signe de $\arctan x$ *est celui de* x*, et puisque* $\dfrac{\pi}{4} > 0$ *alors nécessairement* $x > 0$.

   Soit $x \in \,]0, +\infty[$.

$$0 < \arctan x < \frac{\pi}{2} \quad et \quad -\frac{\pi}{4} < \frac{\pi}{4} - \arctan x < \frac{\pi}{2}$$

   donc

$$\begin{aligned}
(E) &\iff \arctan 2x = \frac{\pi}{4} - \arctan x \\
&\iff 2x = \tan\left(\frac{\pi}{4} - \arctan x\right) \\
&\iff 2x = \frac{\tan\left(\frac{\pi}{4}\right) - \tan(\arctan x)}{1 + \tan\left(\frac{\pi}{4}\right) . \tan(\arctan x)} \\
&\iff 2x = \frac{1-x}{1+x} \\
&\iff 2x - \left(\frac{1-x}{1+x}\right) = 0 \\
&\iff 2x^2 + 3x - 1 = 0
\end{aligned}$$

   On obtient comme solutions : $\dfrac{\sqrt{17}-3}{4}$ *et* $\dfrac{-\sqrt{17}-3}{4}$*. On ne garde que la solution positive. Donc*

$$S = \left\{\frac{\sqrt{17}-3}{4}\right\}.$$

7. *L'équation* (E_7) *est définie dans* $\mathbb{R}$.

   On sait que le signe de $\arctan x$ *est celui de* x*, et puisque* $\dfrac{\pi}{2} > 0$ *alors nécessairement* $x > 0$.

   Soit $x \in \,]0, +\infty[$.

   On a $\arctan x + \arctan\left(\dfrac{1}{x}\right) = \dfrac{\pi}{2}$*, donc l'équation* (E_7) *est équivalente à :*

$$\arctan(x-1) = \arctan\frac{1}{x} \iff x - 1 = \frac{1}{x} \iff x^2 - x - 1 = 0$$

   On obtient comme solutions : $\dfrac{\sqrt{5}+1}{2}$ *et* $\dfrac{1-\sqrt{5}}{2}$*. On ne garde que la solution positive. On a donc :*

$$S = \left\{\frac{1+\sqrt{5}}{2}\right\}.$$

Exercice 3 .

Calculons les limites suivantes :

■ *Calculons :* $\lim\limits_{x\longrightarrow+\infty} \arctan\left(\sqrt{\dfrac{x-1}{x}}\right)$

On a $\lim\limits_{x\longrightarrow+\infty} \sqrt{\dfrac{x-1}{x}} = 1$, *et la fonction* arctan *est continue en* 1, *donc*

$$\lim_{x\longrightarrow+\infty} \arctan\left(\sqrt{\frac{x-1}{x}}\right) = \arctan(1) = \frac{\pi}{4}.$$

■ *Calculons :* $\lim\limits_{x\longrightarrow 2^+} (x-2)\arctan\left(\dfrac{1}{x-2}\right)$.

On a $\lim\limits_{x\longrightarrow 2^+} \dfrac{1}{x-2} = +\infty$, *et* $\lim\limits_{X\longrightarrow+\infty} \arctan(X) = \dfrac{\pi}{2}$, *et on a* $\lim\limits_{x\longrightarrow 2^+} (x-2) = 0$. *Donc*

$$\lim_{x\longrightarrow 2^+} (x-2)\arctan\left(\frac{1}{x-2}\right) = 0.$$

■ *Calculons :* $\lim\limits_{x\longrightarrow+\infty} \dfrac{\arctan(5x^2+x+1)}{x}$.

On a $\lim\limits_{x\longrightarrow+\infty} 5x^2+x+1 = +\infty$, *et* $\lim\limits_{X\longrightarrow+\infty} \arctan X = \dfrac{\pi}{2}$, *et on a* $\lim\limits_{x\longrightarrow+\infty} \dfrac{1}{x} = 0$. *Donc*

$$\lim_{x\longrightarrow+\infty} \frac{\arctan(5x^2+x+1)}{x} = 0.$$

■ *Calculons :* $\lim\limits_{x\longrightarrow\frac{\pi}{2}^+} \arctan(\tan x)$.

On a $\lim\limits_{x\longrightarrow\frac{\pi}{2}^+} \tan x = -\infty$, *et* $\lim\limits_{X\longrightarrow-\infty} \arctan X = \frac{-\pi}{2}$. *Donc*

$$\lim_{x\longrightarrow\frac{\pi}{2}^+} \arctan(\tan x) = -\frac{\pi}{2}.$$

■ *Calculons :* $\lim\limits_{x\longrightarrow+\infty} x\arctan(\sqrt{x}) - \dfrac{\pi}{2}x$

$$\lim_{x\longrightarrow+\infty} x\arctan\left(\sqrt{x}\right) - \frac{\pi}{2}x = \lim_{x\longrightarrow+\infty} x\left(\arctan\left(\sqrt{x}\right) - \frac{\pi}{2}\right)$$

Soit $x \in]0,+\infty[$, *on a :* $x\arctan(\sqrt{x}) - \dfrac{\pi}{2}x = x\left(\arctan(\sqrt{x}) - \dfrac{\pi}{2}\right)$ *et comme*

$$\arctan\left(\sqrt{x}\right) + \arctan\left(\frac{1}{\sqrt{x}}\right) = \frac{\pi}{2}$$

donc : $\arctan(\sqrt{x}) - \dfrac{\pi}{2} = -\arctan\left(\dfrac{1}{\sqrt{x}}\right)$, *d'où*

$$\begin{aligned}
\lim_{x\longrightarrow+\infty} -x\arctan\left(\frac{1}{\sqrt{x}}\right) &= \lim_{x\longrightarrow+\infty} -x\frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{\frac{1}{\sqrt{x}}} \times \frac{1}{\sqrt{x}} \\
&= \lim_{x\longrightarrow+\infty} (-\sqrt{x}) \times \frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{\frac{1}{\sqrt{x}}}
\end{aligned}$$

On a $\displaystyle\lim_{x\longrightarrow+\infty}\frac{1}{\sqrt{x}} = 0$, *et* $\displaystyle\lim_{X\longrightarrow0}\frac{\arctan X}{X} = 1$. *Donc* $\displaystyle\lim_{x\longrightarrow+\infty}(-\sqrt{x}) \times \frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{\frac{1}{\sqrt{x}}} =$
$-\infty$
c'est-à-dire $\displaystyle\lim_{x\longrightarrow+\infty} x\arctan(\sqrt{x}) - \frac{\pi}{2}x = -\infty$.

■ *Calculons* : $\displaystyle\lim_{x\longrightarrow0^+}\frac{1}{x}\arctan\left(\frac{\sqrt{x}}{x+1}\right)$

$$\begin{aligned}
\lim_{x\longrightarrow0^+}\frac{1}{x}\arctan\left(\frac{\sqrt{x}}{x+1}\right) &= \lim_{x\longrightarrow0^+}\frac{1}{x}\frac{\arctan\left(\frac{\sqrt{x}}{x+1}\right)}{\frac{\sqrt{x}}{x+1}} \times \frac{\sqrt{x}}{x+1} \\
&= \lim_{x\longrightarrow0^+}\frac{1}{\sqrt{x}^2} \times \frac{\sqrt{x}}{x+1} \times \frac{\arctan\left(\frac{\sqrt{x}}{x+1}\right)}{\frac{\sqrt{x}}{x+1}} \\
&= \lim_{x\longrightarrow0^+}\frac{1}{\sqrt{x}} \times \frac{1}{x+1}\frac{\arctan\left(\frac{\sqrt{x}}{x+1}\right)}{\frac{\sqrt{x}}{x+1}}
\end{aligned}$$

On pose $X = \dfrac{\sqrt{x}}{x+1}$ *donc* $\displaystyle\lim_{x\longrightarrow0^+}\frac{\arctan\left(\frac{\sqrt{x}}{x+1}\right)}{\frac{\sqrt{x}}{x+1}} = \lim_{X\longrightarrow0^+}\frac{\arctan(X)}{X} = 1$. *On a*

$\displaystyle\lim_{x\longrightarrow0^+}\frac{1}{\sqrt{x}} = +\infty$ *et* $\displaystyle\lim_{x\longrightarrow0^+}\frac{1}{x+1} = 1$ *donc* $\displaystyle\lim_{x\longrightarrow0^+}\frac{1}{\sqrt{x}} \times \frac{1}{x+1}\frac{\arctan\left(\frac{\sqrt{x}}{x+1}\right)}{\frac{\sqrt{x}}{x+1}} = +\infty$

d'où

$$\lim_{x\longrightarrow 0^+}\frac{1}{x}\arctan\left(\frac{\sqrt{x}}{x+1}\right)=+\infty$$

■ *Calculons :* $\lim\limits_{x\longrightarrow+\infty}\sqrt{x^2+1}.\arctan(x)-\frac{\pi}{2}x$

Soit $x\in\,]0,+\infty[\,,$ *on a* $\arctan(x)+\arctan\left(\dfrac{1}{x}\right)=\dfrac{\pi}{2}$ *c'est-à-dire :* $\arctan(x)=\dfrac{\pi}{2}-\arctan\left(\dfrac{1}{x}\right)$. *Donc*

$$\begin{aligned}
\lim_{x\longrightarrow+\infty}\sqrt{x^2+1}.\arctan(x)-\frac{\pi}{2}x &= \lim_{x\longrightarrow+\infty}\sqrt{x^2+1}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{x}\right)\right)-\frac{\pi}{2}x\\
&= \lim_{x\longrightarrow+\infty}\frac{\pi}{2}\sqrt{x^2+1}-\sqrt{x^2+1}.\frac{\arctan\left(\frac{1}{x}\right)}{\frac{1}{x}}\times\frac{1}{x}-\frac{\pi}{2}x\\
&= \lim_{x\longrightarrow+\infty}\frac{\pi}{2}\sqrt{x^2+1}-\frac{\pi}{2}x-\sqrt{1+\frac{1}{x^2}}.\frac{\arctan\left(\frac{1}{x}\right)}{\frac{1}{x}}\\
&= \lim_{x\longrightarrow+\infty}\frac{\pi}{2}x\left(\sqrt{1+\frac{1}{x^2}}-1\right)-\sqrt{1+\frac{1}{x^2}}.\frac{\arctan\left(\frac{1}{x}\right)}{\frac{1}{x}}\\
&= \lim_{x\longrightarrow+\infty}\frac{\pi}{2}x\left(\frac{1}{x^2\left(\sqrt{1+\frac{1}{x^2}}+1\right)}\right)-\sqrt{1+\frac{1}{x^2}}.\frac{\arctan\left(\frac{1}{x}\right)}{\frac{1}{x}}\\
&= \lim_{x\longrightarrow+\infty}\frac{\pi}{2x\left(\sqrt{1+\frac{1}{x^2}}+1\right)}-\sqrt{1+\frac{1}{x^2}}.\frac{\arctan\left(\frac{1}{x}\right)}{\frac{1}{x}}
\end{aligned}$$

Car $\lim\limits_{x\longrightarrow+\infty}\dfrac{1}{x}=0$ *et on pose* $X=\dfrac{1}{x}$ *donc* $\lim\limits_{X\longrightarrow 0}\dfrac{\arctan X}{X}=0$ *et* $\lim\limits_{x\longrightarrow+\infty}\dfrac{\pi}{2x\left(\sqrt{1+\frac{1}{x^2}}+1\right)}=0$ *et* $\lim\limits_{x\longrightarrow+\infty}\sqrt{1+\dfrac{1}{x^2}}=1$ *d'où*

$$\lim_{x\longrightarrow+\infty}\sqrt{x^2+1}.\arctan(x)-\frac{\pi}{2}x=1$$

■ *Calculons :* $\lim\limits_{x\longrightarrow 1^+} \dfrac{x-2\sqrt{\arctan x-\dfrac{\pi}{4}}-1}{x-1}$

$$\begin{aligned}
\lim_{x\longrightarrow 1^+} \frac{x-2\sqrt{\arctan x-\dfrac{\pi}{4}}-1}{x-1} &= \lim_{x\longrightarrow 1^+} \frac{x-1-2\sqrt{\arctan x-\frac{\pi}{4}}}{x-1} \\
&= \lim_{x\longrightarrow 1^+} 1-2\sqrt{\frac{\arctan x-\frac{\pi}{4}}{(x-1)^2}}
\end{aligned}$$

Soit $x \in \left]1,+\infty\right[$. *On a* $\dfrac{\pi}{4} < \arctan(x) < \dfrac{\pi}{2}$ *donc* $0 < \arctan x - \dfrac{\pi}{4} < \dfrac{\pi}{4}$. *D'où :* $\arctan x - \dfrac{\pi}{4} \in \left]\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right[$ *donc*

$$\begin{aligned}
\tan\left(\arctan x-\frac{\pi}{4}\right) &= \frac{\tan(\arctan x)-\tan\dfrac{\pi}{4}}{1+\tan(\arctan x)\,.\tan\dfrac{\pi}{4}} \\
&= \frac{x-1}{1+x}
\end{aligned}$$

on déduit que

$$\arctan x-\frac{\pi}{4} = \arctan\frac{x-1}{1+x}.$$

Donc

$$\begin{aligned}
\lim_{x\longrightarrow 1^+} 1-2\sqrt{\frac{\arctan x-\dfrac{\pi}{4}}{(x-1)^2}} &= \lim_{x\longrightarrow 1^+} 1-2\sqrt{\frac{\arctan\left(\dfrac{x-1}{1+x}\right)}{(x-1)^2}} \\
&= \lim_{x\longrightarrow 1^+} 1-2\sqrt{\frac{\arctan\left(\dfrac{x-1}{1+x}\right)}{\dfrac{x-1}{1+x}} \times \frac{x-1}{(1+x)(x-1)^2}} \\
&= \lim_{x\longrightarrow 1^+} 1-2\sqrt{\frac{\arctan\left(\dfrac{x-1}{1+x}\right)}{\dfrac{x-1}{1+x}} \times \frac{1}{(x^2-1)}}
\end{aligned}$$

On a $\lim\limits_{x\longrightarrow 1^+} \dfrac{x-1}{1+x} = 0$, *on pose* $X = \dfrac{x-1}{1+x}$ *donc* $\lim\limits_{X\longrightarrow 0^+} \dfrac{\arctan(X)}{X} = 1$ *et* $\lim\limits_{x\longrightarrow 1^+}$

$$\frac{x-1}{1+x}=+\infty \textit{ donc } \lim_{x\longrightarrow 1^+} 1-2\sqrt{\frac{\arctan\left(\frac{x-1}{1+x}\right)}{\frac{x-1}{1+x}}\times\frac{1}{(x^2-1)}}=-\infty \textit{ d'où}$$

$$\lim_{x\longrightarrow 1^+}\frac{x-2\sqrt{\arctan x-\frac{\pi}{4}}-1}{x-1}=-\infty$$

Exercice 4 *On simplifie l'expression de la fonction f dans chacun des cas suivants :*

■ $f(x)=\arctan\left(\dfrac{x^2-1}{2x}\right)$.

La fonction f est définie sur $\mathbb{R}^*$.

Soit $x\in\mathbb{R}^*$ il existe un unique $\alpha\in\left]\dfrac{-\pi}{2},\dfrac{\pi}{2}\right[$ tel que $\tan\alpha=x$ et $\alpha\neq 0$, d'où $\alpha=\arctan x$.

$$\begin{aligned}
\frac{x^2-1}{2x} &= \frac{\tan^2\alpha-1}{2\tan\alpha}\\
&= \frac{1}{\dfrac{2\tan\alpha}{\tan^2\alpha-1}}\\
&= \frac{-1}{\dfrac{2\tan\alpha}{1-\tan^2\alpha}}\\
&= \frac{-1}{\tan 2\alpha}\\
&= \tan\left(\frac{\pi}{2}+2\alpha\right)
\end{aligned}$$

Donc : $f(x)=\arctan\left(\tan\left(\dfrac{\pi}{2}+2\alpha\right)\right)$.

On a

$$\begin{aligned}
\begin{cases}\dfrac{-\pi}{2}<\alpha<\dfrac{\pi}{2}\\ \alpha\neq 0\end{cases} &\Longleftrightarrow \begin{cases}-\pi<2\alpha<\pi\\ 2\alpha\neq 0\end{cases}\\
&\Longleftrightarrow \begin{cases}-\pi+\dfrac{\pi}{2}<\dfrac{\pi}{2}+2\alpha<\dfrac{\pi}{2}+\pi\\ \dfrac{\pi}{2}+2\alpha\neq\dfrac{\pi}{2}\end{cases}\\
&\Longleftrightarrow \begin{cases}\dfrac{-\pi}{2}<\dfrac{\pi}{2}+2\alpha<\dfrac{3\pi}{2}\\ \dfrac{\pi}{2}+2\alpha\neq\dfrac{\pi}{2}\end{cases}\\
&\Longleftrightarrow \frac{-\pi}{2}<\frac{\pi}{2}+2\alpha<\frac{\pi}{2} \textit{ ou } \frac{\pi}{2}<\frac{\pi}{2}+2\alpha<\frac{3\pi}{2}
\end{aligned}$$

♣ Si $\dfrac{-\pi}{2} < \dfrac{\pi}{2} + 2\alpha < \dfrac{\pi}{2}$, *alors* $\dfrac{-\pi}{2} < \alpha < 0$, *ceci signifie que* $x < 0$, *donc*

$$f(x) = \arctan\left(\tan\left(\frac{\pi}{2} + 2\alpha\right)\right) = \frac{\pi}{2} + 2\alpha = \frac{\pi}{2} + 2\arctan x$$

♣ Si $\dfrac{\pi}{2} < \dfrac{\pi}{2} + 2\alpha < \dfrac{3\pi}{2}$, *alors* $0 < \alpha < \dfrac{\pi}{2}$, *ceci signifie que* $x > 0$, *donc*

$$\frac{\pi}{2} < \frac{\pi}{2} + 2\alpha < \frac{3\pi}{2} \iff -\frac{\pi}{2} < \left(\frac{\pi}{2} + 2\alpha\right) - \pi < \frac{\pi}{2}$$

et comme $\tan\left(\dfrac{\pi}{2} + 2\alpha\right) = \tan\left(\left(\dfrac{\pi}{2} + 2\alpha\right) - \pi\right)$, *donc*

$$\begin{aligned} f(x) &= \arctan\left(\tan\left(\frac{\pi}{2} + 2\alpha\right)\right) \\ &= \arctan\left(\tan\left(\left(\frac{\pi}{2} + 2\alpha\right) - \pi\right)\right) \\ &= \frac{\pi}{2} + 2\alpha - \pi \\ &= \frac{-\pi}{2} + 2\arctan x \end{aligned}$$

d'où on obtient

$$\begin{cases} f(x) = -\dfrac{\pi}{2} + 2\arctan x & , x > 0 \\ f(x) = \dfrac{\pi}{2} + 2\arctan x & , x < 0 \end{cases}$$

■ $f(x) = \arctan\left(\dfrac{1+x}{1-x}\right)$.

La fonction f *est définie sur* $]-\infty, 1[\cup]1, +\infty[$.

Soit $x \in D_f$ *alors il existe un unique élément* $\alpha \in \left]\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right[$ *tel que* $\tan\alpha = x$ *et* $\alpha \neq \dfrac{\pi}{4}$, *d'où* $\alpha = \arctan x$.

$$\begin{aligned} \frac{1+x}{1-x} &= \frac{1+\tan\alpha}{1-\tan\alpha} \\ &= \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} \\ &= \frac{\sqrt{2}\sin\left(\frac{\pi}{4} + \alpha\right)}{\sqrt{2}\cos\left(\frac{\pi}{4} + \alpha\right)} \\ &= \tan\left(\frac{\pi}{4} + \alpha\right) \end{aligned}$$

Donc $f(x) = \arctan\left(\tan\left(\dfrac{\pi}{4} + \alpha\right)\right)$.

On a

$$\begin{cases} \dfrac{-\pi}{2} < \alpha < \dfrac{\pi}{2} \\ \\ \alpha \neq \dfrac{\pi}{4} \end{cases} \iff \begin{cases} \dfrac{-\pi}{4} < \alpha + \dfrac{\pi}{4} < \dfrac{3\pi}{4} \\ \\ \alpha + \dfrac{\pi}{4} \neq \dfrac{\pi}{2} \end{cases}$$

$$\iff \frac{-\pi}{4} < \alpha + \frac{\pi}{4} < \frac{\pi}{2} \ \text{ou}\ \frac{\pi}{2} < \alpha + \frac{\pi}{4} < \frac{3\pi}{4}$$

♣ Si $\frac{-\pi}{4} < \alpha + \frac{\pi}{4} < \frac{\pi}{2}$, *alors* $\frac{-\pi}{2} < \alpha < \frac{\pi}{4}$, *donc* $x < 1$. *D'où*

$$f(x) = \arctan\left(\tan\left(\frac{\pi}{4} + \alpha\right)\right) = \frac{\pi}{4} + \alpha = \frac{\pi}{4} + \arctan x.$$

C'est-à-dire

$$(\forall x \in]-\infty, 1[), \quad f(x) = \frac{\pi}{4} + \arctan x$$

♣ Si $\frac{\pi}{2} < \alpha + \frac{\pi}{4} < \frac{3\pi}{4}$, *alors* $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$, *donc* $x > 1$, *et puisque*

$$\begin{aligned} \frac{\pi}{2} \quad < \quad & \alpha + \frac{\pi}{4} < \frac{3\pi}{4} \\ \iff \quad & \frac{\pi}{2} - \pi < \left(\alpha + \frac{\pi}{4}\right) - \pi < \frac{3\pi}{4} - \pi \\ \iff \quad & \frac{-\pi}{2} < \left(\alpha + \frac{\pi}{4}\right) - \pi < \frac{-\pi}{4} \end{aligned}$$

et $\tan\left(\frac{\pi}{4} + \alpha\right) = \tan\left(\left(\frac{\pi}{4} + \alpha\right) - \pi\right)$. *D'où*

$$\begin{aligned} f(x) &= \arctan\left(\tan\left(\frac{\pi}{4} + \alpha\right)\right) \\ &= \arctan\left(\tan\left(\left(\frac{\pi}{4} + \alpha\right) - \pi\right)\right) \\ &= \frac{\pi}{4} + \alpha - \pi \\ &= \frac{-3\pi}{4} + \arctan x \end{aligned}$$

C'est-à-dire

$$(\forall x \in]1, +\infty[), \quad f(x) = \frac{-3\pi}{4} + \arctan x$$

Par suite

$$\begin{cases} f(x) = \dfrac{\pi}{4} + \arctan x \quad , x < 1 \\ \\ f(x) = \dfrac{-3\pi}{4} + \arctan x \ , x > 1 \end{cases}$$

■ $f(x) = \arctan\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$.

La fonction f est définie sur $\mathbb{R}^*$.

Soit $x \in \mathbb{R}^*$, il existe un unique $\alpha \in \left]\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right[$ tel que $\tan\alpha = x$ et $\alpha \neq 0$, d'où $\alpha = \arctan x$.

$$\begin{aligned}
\frac{\sqrt{1+x^2}-1}{x} &= \frac{\sqrt{1+\tan^2\alpha}-1}{\tan\alpha} \\
&= \frac{\sqrt{\dfrac{1}{\cos^2\alpha}}-1}{\tan\alpha} \\
&= \frac{\dfrac{1}{\cos\alpha}-1}{\tan\alpha} \\
&= \frac{1-\cos\alpha}{\sin\alpha} \\
&= \frac{1-\cos\left(2\times\dfrac{\alpha}{2}\right)}{\sin\left(2\times\dfrac{\alpha}{2}\right)} \\
&= \frac{\cos^2\left(\dfrac{\alpha}{2}\right)+\sin^2\left(\dfrac{\alpha}{2}\right)-\left(\cos^2\left(\dfrac{\alpha}{2}\right)-\sin^2\left(\dfrac{\alpha}{2}\right)\right)}{2\sin\left(\dfrac{\alpha}{2}\right).\cos\left(\dfrac{\alpha}{2}\right)} \\
&= \frac{2\sin^2\left(\dfrac{\alpha}{2}\right)}{2\sin\left(\dfrac{\alpha}{2}\right).\cos\left(\dfrac{\alpha}{2}\right)} \\
&= \frac{\sin\left(\dfrac{\alpha}{2}\right)}{\cos\left(\dfrac{\alpha}{2}\right)} \\
&= \tan\left(\frac{\alpha}{2}\right)
\end{aligned}$$

Donc : $f(x) = \arctan\left(\tan\left(\dfrac{\alpha}{2}\right)\right)$.

On a

$$\begin{cases} \dfrac{-\pi}{2} < \alpha < \dfrac{\pi}{2} \\ \alpha \neq 0 \end{cases} \iff \begin{cases} \dfrac{-\pi}{4} < \dfrac{\alpha}{2} < \dfrac{\pi}{4} \\ \dfrac{\alpha}{2} \neq 0 \end{cases}$$

d'où $f(x) = \arctan\left(\tan\left(\dfrac{\alpha}{2}\right)\right) = \dfrac{\alpha}{2} = \dfrac{1}{2}\arctan x$.

Donc

$$(\forall x \in \mathbb{R}^*), \quad f(x) = \frac{1}{2}\arctan x$$

■ $f(x) = \arctan\left(\sqrt{1+x^2} - x\right)$.

La fonction f *est définie sur* $\mathbb{R}$.

Soit $x \in \mathbb{R}$, *il existe un unique* $\alpha \in \left]\frac{-\pi}{2}, \frac{\pi}{2}\right[$ *tel que* $\tan\alpha = x$, *d'où* $\alpha = \arctan x$.

$$\begin{aligned}
\sqrt{1+x^2} - x &= \sqrt{1+\tan^2\alpha} - \tan\alpha \\
&= \sqrt{1+\frac{\sin^2\alpha}{\cos^2\alpha}} - \frac{\sin\alpha}{\cos\alpha} \\
&= \sqrt{\frac{1}{\cos^2\alpha}} - \frac{\sin\alpha}{\cos\alpha} \\
&= \frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha} \\
&= \frac{1-\sin\alpha}{\cos\alpha} \\
&= \frac{1-\sin\left(2\times\frac{\alpha}{2}\right)}{\cos\left(2\times\frac{\alpha}{2}\right)} \\
&= \frac{1-2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)}{\cos\left(2\times\frac{\alpha}{2}\right)} \\
&= \frac{\cos^2\left(\frac{\alpha}{2}\right)+\sin^2\left(\frac{\alpha}{2}\right)-2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)}{\cos^2\left(\frac{\alpha}{2}\right)-\sin^2\left(\frac{\alpha}{2}\right)} \\
&= \frac{\cos^2\left(\frac{\alpha}{2}\right)-2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)+\sin^2\left(\frac{\alpha}{2}\right)}{\cos^2\left(\frac{\alpha}{2}\right)-\sin^2\left(\frac{\alpha}{2}\right)} \\
&= \frac{\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)^2}{\left(\cos\left(\frac{\alpha}{2}\right)+\sin\left(\frac{\alpha}{2}\right)\right)\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)} \\
&= \frac{\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)+\sin\left(\frac{\alpha}{2}\right)} \\
&= \frac{\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)} \\
&= \tan\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)
\end{aligned}$$

Donc : $f(x) = \arctan\left(\tan\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right)$.

On a

$$\begin{aligned}-\frac{\pi}{2} &< \alpha < \frac{\pi}{2}\\ &\Longleftrightarrow \frac{-\pi}{4} < \frac{-\alpha}{2} < \frac{\pi}{4}\\ &\Longleftrightarrow 0 < \frac{\pi}{4} - \frac{\alpha}{2} < \frac{\pi}{2}\end{aligned}$$

d'où : $f(x) = \arctan\left(\tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\right) = \frac{\pi}{4} - \frac{\alpha}{2} = \frac{\pi}{4} - \frac{1}{2}\arctan x.$

Donc

$$(\forall x \in \mathbb{R}),\ f(x) = \frac{\pi}{4} - \frac{1}{2}\arctan x.$$

FIN

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