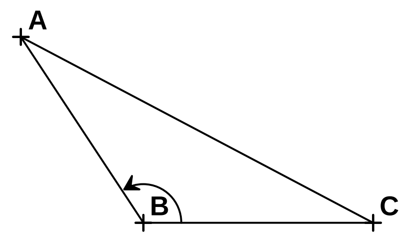
www.etude-generale.com **TCSI**
Matière : Mathématiques
Professeur : Yahya MATIOUI

Correction du devoir Maison N8

Exercice 1 .

1. *Construction du triangle ABC :*



2. *On cherche les mesures des angles orientés :* $\left(\widehat{\overrightarrow{AB}, \overrightarrow{AC}}\right)$, $\left(\widehat{\overrightarrow{AB}, \overrightarrow{BC}}\right)$ *et* $\left(\widehat{\overrightarrow{CA}, \overrightarrow{BC}}\right)$.

   ♣ *Le triangle ABC est isocèle en B et comme* $\widehat{ABC} = \frac{2\pi}{3}$ *donc* $\widehat{BAC} = \frac{\pi}{6}$, *et comme l'angle* $\left(\widehat{\overrightarrow{AB}, \overrightarrow{AC}}\right)$ *est orienté positivement donc* $\frac{\pi}{6}$ *est la mesure principale de l'angle orienté* $\left(\widehat{\overrightarrow{AB}, \overrightarrow{AC}}\right)$.

   ♣ *On a* $\left(\overline{\overrightarrow{AB}, \overrightarrow{BC}}\right) \equiv \left(\overline{-\overrightarrow{BA}, \overrightarrow{BC}}\right) [2\pi]$ *et comme* $\left(\overline{-\overrightarrow{BA}, \overrightarrow{BC}}\right) \equiv \pi - \left(\overline{\overrightarrow{BC}, \overrightarrow{BA}}\right) [2\pi]$ *c'est-à-dire* $\left(\overline{-\overrightarrow{BA}, \overrightarrow{BC}}\right) \equiv \pi - \frac{2\pi}{3} [2\pi]$ *d'où* $\left(\overline{-\overrightarrow{BA}, \overrightarrow{BC}}\right) \equiv \frac{\pi}{3} [2\pi]$ *donc*

$$\left(\overline{\overrightarrow{AB}, \overrightarrow{BC}}\right) \equiv \frac{\pi}{3} [2\pi]$$

   d'où $\frac{\pi}{3}$ *est la mesure principale de l'angle orienté* $\left(\widehat{\overrightarrow{AB}, \overrightarrow{BC}}\right)$.

   ♣ *On a* $\left(\overline{\overrightarrow{CA}, \overrightarrow{BC}}\right) \equiv \left(\overline{\overrightarrow{CA}, -\overrightarrow{CB}}\right) [2\pi]$ *et comme*

$$\left(\overline{\overrightarrow{CA}, -\overrightarrow{CB}}\right) \equiv \pi + \left(\overline{\overrightarrow{CA}, \overrightarrow{CB}}\right) [2\pi]$$

On a $\widehat{BCA}+\widehat{ABC}+\widehat{BAC}=\pi$ *et comme le triangle* ABC *est isocèle et* $\widehat{ABC}=\frac{2\pi}{3}$ *donc:* $\widehat{BCA}=\frac{\pi}{6}$ *comme l'angle* $\left(\widehat{\overrightarrow{CA},\overrightarrow{CB}}\right)$ *est orienté positivement donc* $\left(\overline{\overrightarrow{CA},\overrightarrow{CB}}\right)\equiv\frac{\pi}{6}\,[2\pi]$ *d'où on obtient*

$$\begin{aligned}\left(\overline{\overrightarrow{CA},-\overrightarrow{CB}}\right) &\equiv \pi+\frac{\pi}{6}\,[2\pi]\\ &\equiv \frac{7\pi}{6}\,[2\pi]\end{aligned}$$

or $\frac{7\pi}{6}\notin\,]-\pi,\pi]$ *et* $\frac{7\pi}{6}=2\pi-\frac{5\pi}{6}$ *donc* $\left(\overline{\overrightarrow{CA},-\overrightarrow{CB}}\right)\equiv\frac{-5\pi}{6}\,[2\pi]$ *d'oú*

$$\left(\overline{\overrightarrow{CA},\overrightarrow{BC}}\right)\equiv\frac{-5\pi}{6}\,[2\pi]$$

d'où $\frac{-5\pi}{6}$ *est la mesure principale de l'angle orienté* $\left(\widehat{\overrightarrow{CA},\overrightarrow{BC}}\right)$.

Exercice 2 .

Soient a, b *et* α *des réels tels que* $a+b\neq 0$ *et* $\alpha\neq\frac{\pi}{2}+k\pi$ *avec* $k\in\mathbb{Z}$. *On pose*

$$x=\frac{a}{\cos\alpha}+b\tan\alpha \quad \textit{et} \quad y=\frac{b}{\cos\alpha}+a\tan\alpha$$

1. *Montrons que :* $\frac{x+y}{a+b}=\frac{1+\sin\alpha}{\cos\alpha}$

   On a

$$\begin{aligned}\frac{x+y}{a+b} &= \frac{\frac{a}{\cos\alpha}+b\tan\alpha+\frac{b}{\cos\alpha}+a\tan\alpha}{a+b}\\ &= \frac{\frac{1}{\cos\alpha}(a+b)+\tan\alpha\,(a+b)}{a+b}\\ &= \frac{(a+b)\left(\frac{1}{\cos\alpha}+\tan\alpha\right)}{a+b}\\ &= \frac{1}{\cos\alpha}+\tan\alpha\\ &= \frac{1}{\cos\alpha}+\frac{\sin\alpha}{\cos\alpha}\\ &= \frac{1+\sin\alpha}{\cos\alpha}\end{aligned}$$

2. Si $x+y=2(a+b)$ *et comme* $\frac{x+y}{a+b}=\frac{1+\sin\alpha}{\cos\alpha}$ *alors on obtient* $2=\frac{1+\sin\alpha}{\cos\alpha}$ *donc* $2\cos\alpha=1+\sin\alpha$.

On sait que $\cos^2\alpha = 1-\sin^2\alpha$ *et comme* $\alpha \in \left]\dfrac{-\pi}{2},\dfrac{\pi}{2}\right[$ *donc* $\cos\alpha = \sqrt{1-\sin^2\alpha}$ *d'où* $2\sqrt{1-\sin^2\alpha} = 1+\sin\alpha$ *qui est équivaut à*

$$\begin{array}{rcl}
4\left(1-\sin^2\alpha\right) & = & (1+\sin\alpha)^2 \\
 & \iff & 4\,(1-\sin\alpha)\,(1+\sin\alpha)-(1+\sin\alpha)^2=0 \\
 & \iff & (1+\sin\alpha)\,[4\,(1-\sin\alpha)-(1+\sin\alpha)]=0 \\
 & \iff & (1+\sin\alpha)\,(4-4\sin\alpha-1-\sin\alpha)=0 \\
 & \iff & (1+\sin\alpha)\,(3-5\sin\alpha)=0 \\
 & \iff & 1+\sin\alpha=0 \;\; ou \;\; 3-5\sin\alpha=0 \\
 & \iff & \sin\alpha=-1 \;\; ou \;\; \sin\alpha=\dfrac{3}{5}
\end{array}$$

comme $\alpha \in \left]-\dfrac{\pi}{2},\dfrac{\pi}{2}\right[$ *alors* $\sin\alpha \neq -1$ *donc*

$$\sin\alpha = \frac{3}{5}$$

On cherche $\cos\alpha$

On a $\cos^2\alpha = 1-\sin^2\alpha$ *donc* $\cos^2\alpha = 1-\dfrac{9}{25}=\dfrac{16}{25}$ *d'où* $\cos\alpha=\dfrac{4}{5}$.

Exercice 3 .

On a $\sin\dfrac{\pi}{5} = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$,

♣ *calculons* $\sin\dfrac{101\pi}{5}$.

On a

$$\sin\frac{101\pi}{5} = \sin\left(\frac{100\pi}{5}+\frac{\pi}{5}\right) = \sin\left(20\pi+\frac{\pi}{5}\right) = \sin\frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

♣ *calculons* $\sin\dfrac{4\pi}{5}$.

On a

$$\sin\frac{4\pi}{5} = \sin\left(\frac{5\pi}{5}-\frac{\pi}{5}\right) = \sin\left(\pi-\frac{\pi}{5}\right) = \sin\frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

♣ *calculons* $\sin\dfrac{-49\pi}{5}$.

On a

$$\sin\frac{-49\pi}{5} = -\sin\left(\frac{49\pi}{5}\right) = -\sin\left(\frac{50\pi}{5}-\frac{\pi}{5}\right) = -\sin\left(10\pi-\frac{\pi}{5}\right) = -\sin\left(\frac{-\pi}{5}\right) = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

Exercice 4 .

Soient β et c deux réels tels que $\cos\alpha = b$.

$$\begin{aligned}
& \cos(5\pi - \alpha) + \sin\left(\frac{13\pi}{2} + \alpha\right) + \cos(11\pi + \alpha) + \sin\left(\frac{31\pi}{2} + \alpha\right) \\
= \;& \cos(4\pi + \pi - \alpha) + \sin\left(\frac{12\pi}{2} + \frac{\pi}{2} + \alpha\right) + \cos(\pi + 10\pi + \alpha) + \sin\left(\frac{\pi + 30\pi}{2} + \alpha\right) \\
= \;& \cos(\pi - \alpha) + \sin\left(\frac{\pi}{2} + \alpha\right) + \cos(\pi + \alpha) + \sin\left(15\pi + \frac{\pi}{2} + \alpha\right) \\
= \;& \cos(\pi - \alpha) + \sin\left(\frac{\pi}{2} + \alpha\right) + \cos(\pi + \alpha) + \sin\left(\pi + 14\pi + \frac{\pi}{2} + \alpha\right) \\
= \;& -\cos\alpha + \cos\alpha - \cos\alpha - \sin\left(\frac{\pi}{2} + \alpha\right) \\
= \;& -\cos\alpha - \cos\alpha \\
= \;& -b - b \\
= \;& -2b
\end{aligned}$$

FIN

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