Correction du devoir Surveillé

Exercice 1 .

On considère deux points A et B d'abscisses curvilignes respectives $\frac{95\pi}{6}$ *et* $\frac{-44\pi}{3}$ *sur* (C) .

1. *On cherche l'abscisse curviligne principale de chacun des points A et B.*

♣ *On a* $\frac{95\pi}{6} = \frac{90\pi + 5\pi}{6} = \frac{90\pi}{6} + \frac{5\pi}{6} = 15\pi + \frac{5\pi}{6} = 16\pi - \pi + \frac{5\pi}{6} = 16\pi - \frac{\pi}{6}$

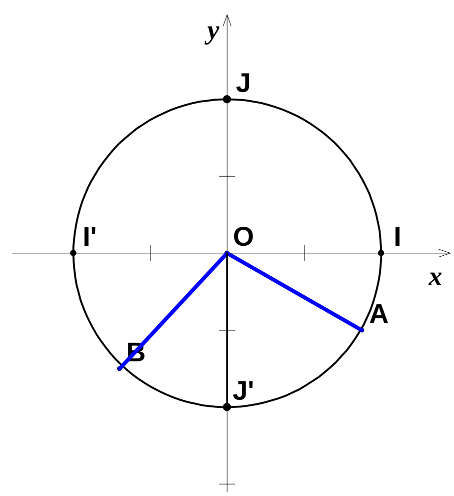
donc $\frac{-\pi}{6}$ *est l'abscisse curviligne principale de* $\frac{95\pi}{6}$.

♣ *On a*

$$\begin{aligned} \frac{-44\pi}{3} &= \frac{-45\pi + \pi}{3} \\ &= \frac{-45\pi}{3} + \frac{\pi}{3} \\ &= -15\pi + \frac{\pi}{3} \\ &= -14\pi - \pi + \frac{\pi}{3} \\ &= -14\pi - \frac{2\pi}{3} \end{aligned}$$

donc $\frac{-2\pi}{3}$ *est l'abscisse curviligne principale de* $\frac{-44\pi}{3}$.

2. *On trace le cercle*



3. **a)** *On cherche la mesure principale de l'angle orienté* $\left(\widehat{\overrightarrow{OA}, \overrightarrow{OB}}\right)$.

On a d'après la relation de chasles

$$\begin{aligned}\left(\overrightarrow{OA}, \overrightarrow{OB}\right) &\equiv \left(\overrightarrow{OA}, \overrightarrow{OI}\right) + \left(\overrightarrow{OI}, \overrightarrow{OB}\right) [2\pi] \\ &\equiv \frac{\pi}{6} - \frac{2\pi}{3} [2\pi] \\ &\equiv \frac{-\pi}{2} [2\pi]\end{aligned}$$

par suite la mesure principale de l'angle $\left(\widehat{\overrightarrow{OA}, \overrightarrow{OB}}\right)$ *est* $\frac{-\pi}{2}$.

b) *Comme* $OA = OB$ *et* $\left(\overrightarrow{OA}, \overrightarrow{OB}\right) \equiv -\frac{\pi}{2} [2\pi]$ *donc le triangle* OAB *est rectangle isocèle en* O.

Exercice 2 .

Soit x *un réel. On pose* $\cos x - \sin x = \alpha$.
Calculons en fonction α *chacun des expressions*

♣ $A = \cos(x) \times \sin(x)$:

On a

$$\begin{aligned}A &= \cos(x) \times \sin(x) \\ &= (\cos(x) - \sin(x))^2 + 3\cos(x).\sin(x) - 1 \\ &= \alpha^2 + 3A - 1\end{aligned}$$

donc $A = \alpha^2 + 3A - 1$ *par suite* $-2A = \alpha^2 - 1$ *d'où :* $A = \frac{1-\alpha^2}{2}$.

♣ $B = \cos(x) \times \sin^2(x) - \sin(x) \times \cos^2(x)$:

$$\begin{aligned}B &= \cos(x) \times \sin^2(x) - \sin(x) \times \cos^2(x) \\ &= \cos(x)\left(1 - \cos^2(x)\right) - \sin(x)\left(1 - \sin^2(x)\right) \\ &= \cos(x) - \cos^3(x) - \sin(x) + \sin^3(x) \\ &= \cos(x) - \sin(x) - \left(\cos^3(x) - \sin^3(x)\right) \\ &= \alpha - (\cos(x) - \sin(x))\left(\cos^2(x) + \cos(x).\sin(x) + \sin^2(x)\right) \\ &= \alpha - \alpha \times \left(1 + \frac{1-\alpha^2}{2}\right) \\ &= \alpha - \alpha - \alpha\left(\frac{1-\alpha^2}{2}\right) \\ &= -\frac{\alpha\left(1-\alpha^2\right)}{2}\end{aligned}$$

Exercice 3 .

Soit θ *un réel tel que* $\theta \in \left]\frac{\pi}{2},\pi\right[$ *et* $\sin\theta = \frac{\sqrt{2}-1}{2}$.

Calculons $\cos\theta$:

On a $\cos^2\theta + \sin^2\theta = 1$ *alors* $\cos^2\theta = 1 - \sin^2\theta$ *donc*

$$\begin{aligned}
\cos^2\theta &= 1 - \left(\frac{\sqrt{2}-1}{2}\right)^2 \\
&= 1 - \frac{\left(\sqrt{2}-1\right)^2}{4} \\
&= \frac{4-\left(\sqrt{2}-1\right)^2}{4} \\
&= \frac{4-\left(2-2\sqrt{2}+1\right)}{4} \\
&= \frac{1+2\sqrt{2}}{4}
\end{aligned}$$

d'où $\cos^2\theta = \frac{1+2\sqrt{2}}{4}$ *par suite* $|\cos\theta| = \frac{\sqrt{1+2\sqrt{2}}}{2}$ *et comme* $\theta \in \left]\frac{\pi}{2},\pi\right[$ *alors* $\cos\theta < 0$ *donc* $|\cos\theta| = -\cos\theta$ *d'où*

$$\cos\theta = -\frac{\sqrt{1+2\sqrt{2}}}{2}$$

Calculons $\tan\theta$:

Soit $\theta \in \left]\frac{\pi}{2},\pi\right[$.

On a

$$\begin{aligned}
\tan\theta &= \frac{\sin\theta}{\cos\theta} \\
&= -\frac{\frac{\sqrt{2}-1}{2}}{\frac{\sqrt{1+2\sqrt{2}}}{2}} \\
&= -\frac{\sqrt{2}-1}{\sqrt{1+2\sqrt{2}}} \\
&= -\frac{\left(\sqrt{2}-1\right)\sqrt{1+2\sqrt{2}}}{1+2\sqrt{2}} \\
&= -\frac{\left(1-2\sqrt{2}\right)\left(\sqrt{2}-1\right)\sqrt{1+2\sqrt{2}}}{-7} \\
&= \frac{\sqrt{2\sqrt{2}+1}\left(3\sqrt{2}-5\right)}{7}
\end{aligned}$$

Exercice 4 .

♠

$$\begin{aligned}
A &= \cos\left(\frac{\pi}{10}\right)\times\sin\left(\frac{3\pi}{5}\right)-\sin\left(\frac{\pi}{10}\right)\times\cos\left(\frac{3\pi}{5}\right)\\
&= \cos\left(\frac{5\pi-4\pi}{10}\right)\times\sin\left(\frac{5\pi-2\pi}{5}\right)-\sin\left(\frac{5\pi-4\pi}{10}\right)\times\cos\left(\frac{5\pi-2\pi}{5}\right)\\
&= \cos\left(\frac{\pi}{2}-\frac{2\pi}{5}\right)\times\sin\left(\pi-\frac{2\pi}{5}\right)-\sin\left(\frac{\pi}{2}-\frac{2\pi}{5}\right)\times\cos\left(\pi-\frac{2\pi}{5}\right)\\
&= \sin\left(\frac{2\pi}{5}\right)\times\sin\left(\frac{2\pi}{5}\right)-\cos\left(\frac{2\pi}{5}\right)\times\left(-\cos\left(\frac{2\pi}{5}\right)\right)\\
&= \sin^2\left(\frac{2\pi}{5}\right)+\cos^2\left(\frac{2\pi}{5}\right)\\
&= 1
\end{aligned}$$

♣

$$\begin{aligned}
B &= 1+\cos\left(\frac{\pi}{7}\right)+\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{3\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{5\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)\\
&= 1+\cos\left(\frac{\pi}{7}\right)+\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{3\pi}{7}\right)+\cos\left(\pi-\frac{3\pi}{7}\right)+\cos\left(\pi-\frac{2\pi}{7}\right)+\cos\left(\pi-\frac{\pi}{7}\right)\\
&= 1+\cos\left(\frac{\pi}{7}\right)+\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{3\pi}{7}\right)-\cos\left(\frac{3\pi}{7}\right)-\cos\left(\frac{2\pi}{7}\right)-\cos\left(\frac{\pi}{7}\right)\\
&= 1
\end{aligned}$$

♣

$$\begin{aligned}
C &= \cos^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\cos^2\left(\frac{5\pi}{8}\right)+\cos^2\left(\frac{7\pi}{8}\right)\\
&= \cos^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\cos^2\left(\frac{4\pi+\pi}{8}\right)+\cos^2\left(\frac{4\pi+3\pi}{8}\right)\\
&= \cos^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\cos^2\left(\frac{\pi}{2}+\frac{\pi}{8}\right)+\cos^2\left(\frac{\pi}{2}+\frac{3\pi}{8}\right)\\
&= \cos^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\sin^2\left(\frac{\pi}{8}\right)+\sin^2\left(\frac{3\pi}{8}\right)\\
&= \cos^2\left(\frac{\pi}{8}\right)+\sin^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\sin^2\left(\frac{3\pi}{8}\right)\\
&= 1+1\\
&= 2
\end{aligned}$$

♣

$$
\begin{aligned}
D &= 1+\sin\left(\frac{\pi}{3}\right)+\sin\left(\frac{2\pi}{3}\right)+\sin\left(\frac{-\pi}{3}\right)+\sin\left(\frac{5\pi}{3}\right)+\cos\left(\frac{11\pi}{4}\right)+\cos\left(\frac{11\pi}{3}\right)\\
&= 1+\sin\left(\frac{\pi}{3}\right)+\sin\left(\frac{2\pi}{3}\right)-\sin\left(\frac{\pi}{3}\right)+\sin\left(\frac{3\pi+2\pi}{3}\right)+\cos\left(\frac{12\pi-\pi}{4}\right)+\cos\left(\frac{12\pi-\pi}{3}\right)\\
&= 1+\sin\left(\frac{2\pi}{3}\right)+\sin\left(\pi+\frac{2\pi}{3}\right)+\cos\left(3\pi-\frac{\pi}{4}\right)+\cos\left(4\pi-\frac{\pi}{3}\right)\\
&= 1+\sin\left(\frac{2\pi}{3}\right)-\sin\left(\frac{2\pi}{3}\right)+\cos\left(\pi-\frac{\pi}{4}\right)+\cos\left(\frac{-\pi}{3}\right)\\
&= 1-\cos\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{3}\right)\\
&= 1-\frac{\sqrt{2}}{2}+\frac{1}{2}\\
&= \frac{3-\sqrt{2}}{2}
\end{aligned}
$$

FIN

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